Mathematical modelling of compressible viscous fluid flow in a male rotor-housing gap of screw machines

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This contribution is devoted to the mathematical modelling of a compressible viscous fluid flow through a 2-D model of the male rotor-housing gap in screw machines. Numerical solution of the nonlinear conservative system of the compressible Navier-Stokes equations is obtained by means of the cell-centred finite volume formulation of the explicit two-step TVD MacCormack scheme proposed by Causon on a structured quadrilateral grid using the own developed numerical code.

1 Introduction

Gas leakage is a phenomenon that has a lot of different features, many of significant importance. Compressor engineers are mostly interested in estimation for the mass flow rate. It has a great influence on the compressor performance, especially with regard to its internal efficiency. Therefore it is necessary to make reasonable estimates for mass flow rates and to investigate the details of the leakage flow. The aim of this contribution is to show one of the possible numerical methods for the computation of a compressible viscous fluid flow through the male rotor-housing gap (the gap between the stator and the head of the male rotor tooth indicated as 5 in Fig. 1) in a screw compressor for the pressure ratio \( p_{01}/p_2 \approx 2 \). It is assumed that the male rotor does not move, that the leakage flow through this gap of 0.1 mm height, Fig. 1, is laminar and that the male rotor-housing gap can be modelled by a two-dimensional bounded domain \( \Omega \subset \mathbb{R}^2 \) with the boundary \( \partial \Omega = \partial \Omega_I \cup \partial \Omega_O \cup \partial \Omega_W \), where \( \partial \Omega_I \) is the inlet and \( \partial \Omega_O \) the outlet of the computational domain \( \Omega \) and \( \partial \Omega_W = \partial \Omega_{WR} \cup \partial \Omega_{WS} \). \( \partial \Omega_{WR} \) are impermeable walls of the computational domain corresponding to the stator \( \partial \Omega_{WS} \) and the head of the male rotor tooth \( \partial \Omega_{WR} \).

2 Mathematical model of a laminar compressible Newtonian fluid flow

Let \((0, T)\) be a time interval. The mathematical model of a laminar compressible fluid flow is described by the nonlinear conservative system of the Navier-Stokes (NS) equations. For 2-D problems it can be written in nondimensional form as

\[
\frac{\partial w}{\partial t} + \frac{\partial f(w)}{\partial x} + \frac{\partial g(w)}{\partial y} = \frac{1}{Re} \left[ \frac{\partial f_V(w)}{\partial x} + \frac{\partial g_V(w)}{\partial y} \right] \quad \text{in} \quad \Omega \times (0, T).
\]

(1)

The vector \( w \) of conservative variables and the inviscid \( f(w) \), \( g(w) \) and viscous \( f_V(w) \), \( g_V(w) \) fluxes are defined as

\[
w = (\varrho, \varrho u, \varrho v, E)^T, \quad f(w) = (\varrho u, \varrho u^2 + p, \varrho u v, (E + p) u)^T, \quad g(w) = (\varrho g, \varrho u g, \varrho v g, (E + p) v)^T, \quad f_V(w) = (0, \tau_{xx}, \tau_{xy}, u \tau_{xx} + v \tau_{xy} - q_x)^T, \quad g_V(w) = (0, \tau_{yz}, \tau_{yy}, u \tau_{yz} + v \tau_{yy} - q_y)^T,
\]

where \( t \) is time, \( x, y \) are Cartesian coordinates, \( \varrho \) is density, \( p \) static pressure, \( E \) total energy per unit volume, \( u, v \) are Cartesian components of the velocity vector \( \mathbf{v} \), \( \tau_{xx}, \tau_{xy}, \tau_{yz}, \tau_{yy} \) are laminar shear stresses and \( q_x, q_y \) are heat flux terms given for a Newtonian fluid, cf. [3]. The external volume forces are neglected in our case.

To close the system of the compressible NS equations (1) it is necessary to specify the equation of state for the perfect gas

\[
p = g r T \equiv (\kappa - 1) \varrho c_v T \equiv (\kappa - 1) \left( \frac{E}{2} + \frac{\varrho}{\kappa} \right)^{2/3}, \quad \text{where} \ T \ \text{is thermodynamic temperature,} \ \tau = c_p - c_v \ \text{is the gas constant per unit mass,} \ c_p \ \text{and} \ c_v \ \text{are the specific heats at constant pressure and volume, respectively and} \ \kappa = 1.4 \ \text{is so-called Poisson’s constant.}
\]

The laminar Prandtl number \( Pr = c_p \eta / k \) is taken to be 0.72 for the calorically perfect gas, where \( \eta \) is molecular viscosity and \( k \) thermal conductivity. The reference Reynolds number is defined as \( Re_{\infty} = \varrho_{\text{ref}} u_{\text{ref}} l_{\text{ref}} / \eta_{\text{ref}} \).

2.1 Non-dimensional boundary conditions

We consider the flow with the reference Reynolds number \( Re_{\infty} = 3900 \). At the inlet \( \partial \Omega_I \), the stagnation pressure \( p_{01} = 1 \), the stagnation temperature \( T_{01} = 1 \), the inlet angle \( \alpha_1, \theta_{\text{in}} = 0 \) and \( \sum_{j=1}^2 \tau_{ij} n_j = 0, i = 1, 2 \) are prescribed. At the outlet \( \partial \Omega_O \), the static pressure \( p_2 = 0.5, \theta_{\text{in}} = 0 \) and \( \sum_{j=1}^2 \tau_{ij} n_j = 0, i = 1, 2 \) are kept. On the solid walls \( \partial \Omega_{WS} \) and \( \partial \Omega_{WR} \), the boundary conditions \( u = 0, v = 0 \) and \( \theta_{\text{in}} = 0 \) are satisfied. \( n \) is the outward unit normal vector to the boundary.

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3 Numerical method

For the discretization of the nonlinear conservative system of the compressible NS equations (1) the cell-centred finite volume (FV) method on a structured quadrilateral grid, cf. [2, 3], is used. Time integration of the inviscid part of the system (1) is performed by using the FV formulation of the explicit two-step TVD MacCormack scheme proposed by Causon, cf. [1]. The approximations of the viscous part of the system (1) are added to the predictor and corrector steps of the MacCormack scheme

\[
\begin{align*}
\omega_{ij}^{n+\frac{1}{2}} &= w_{ij}^n - \frac{\Delta t}{\Omega_{ij}} \sum_{m=1}^{4} \left( f_{m}^{n} S_{m}^{n} + g_{m}^{n} w_{m}^{n} \right) + \frac{\Delta t}{\Omega_{ij}} \operatorname{Visc}(w_{ij}^{n}), \\
\omega_{ij}^{n+1} &= \frac{1}{2} \left\{ w_{ij}^{n} + \omega_{ij}^{n+\frac{1}{2}} - \frac{\Delta t}{\Omega_{ij}} \sum_{m=1}^{4} \left( f_{m}^{n+\frac{1}{2}} S_{m}^{n} + g_{m}^{n+\frac{1}{2}} w_{m}^{n} \right) \right\} + \frac{1}{2} \frac{\Delta t}{\Omega_{ij}} \operatorname{Visc}(w_{ij}^{n+\frac{1}{2}}), \\
(w^{TV})_{ij}^{n+1} &= w_{ij}^{n+1} + d\omega_{ij}^{n+1} + dw_{ij}^{2n},
\end{align*}
\]

where \((TVD)_{ij}^{n+1}\) is the corrected numerical solution at time \(t_{n+1}\), \(\Omega_{ij}\) denotes the face area of the finite volume \(\Omega_{ij}\), the inviscid numerical fluxes \(f_{m}^{n}\) through the edges \(\Gamma_{ij}^{m}\), \(m = 1, \ldots, 4\), of the cell \(\Omega_{ij}\) at time \(t_{n}\) are evaluated as \(f_{1}^{n} = f_{i+1,j}^{n}\), \(f_{2}^{n} = f_{ij+1}^{n}\), \(f_{3}^{n} = f_{i-1,j}^{n}\), \(f_{4}^{n} = f_{ij-1}^{n}\) and at time \(t_{n+\frac{1}{2}}\) as \(f_{1}^{n+\frac{1}{2}} \equiv f_{1}^{n+\frac{1}{2}}\). The numerical fluxes \(g_{m}^{n}\) are computed in the same way, cf. [2]. \(S_{m} = (S_{m}^{n}, S_{m}^{n})^{T}\) are cell side normal vectors to the edges \(\Gamma_{ij}^{m}\), where we designate, cf. [3], \(S_{1} = S_{x+}, S_{2} = S_{y+}, S_{3} = S_{x-}, S_{4} = S_{y-}\). The viscous terms \(\operatorname{Visc}(w_{ij})\) in (2) and (3) are approximated by using a FV version with central differences, cf. [3] for details. The added one-dimensional TVD-type viscosity terms \(d\omega_{ij}^{n+1}\) and \(d\omega_{ij}^{2n}\) in the direction of the change of index \(i\) and \(j\) respectively, are defined by Causon, cf. [1].

4 Numerical results and conclusions

For the laminar flow computation through the 2-D model of the male rotor-housing gap, Fig. 1, in a screw compressor the relatively fine computational grid with \(170 \times 68\) quadrilateral cells is used. In order to resolve the boundary layer with sufficient accuracy, the computational grid is refined in the direction normal to the walls. The isolines of the Mach number in the male rotor-housing gap plotted with \(\Delta M = 0.03\) at times \(t_{1} = 2.331 \cdot 10^{-4}\) s and \(t_{2} = 2.395 \cdot 10^{-4}\) s are shown in Fig. 2.

As a conclusion of this numerical testing it can be deduced that the laminar flow through the male rotor-housing gap in a screw compressor is transonic and non-stationary (see the changes in the shape of the wake after \(6.4 \cdot 10^{-5}\) s, Fig. 2). These results are generally in good agreement with the results obtained by using the software package FLUENT.

Fig. 1 Frontal section of male-1 and female-2 rotors (left) and geometry of the male rotor-housing gap with mesh (right).

Fig. 2 Isolines of the Mach number at times \(t_{1} = 2.331 \cdot 10^{-4}\) s (left) and \(t_{2} = 2.395 \cdot 10^{-4}\) s (right).

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