UNIVERSITY OF WEST BOHEMIA

Applied and Computational Mechanics 3 (2009) 121–132

Applied and Computational Mechanics

Influence of the terminal muffler geometry with three chambers and two tailpipes topology on its attenuation characteristics

R. Matas[a,*], J. Kňourek[a], J. Voldřich[a]

[a] *New Technologies Research Centre, University of West Bohemia, Univerzitní 8, 306 14 Plzeň, Czech Republic*

Received 29 September 2008; received in revised form 25 November 2008

Abstract

Aimed at describing the harmonic acoustic waves passage through an exhaust system, the two-tailpipes terminal muffler may be seen as an acoustic element defined by eight quantities (usually arranged in a 2 by 4 matrix). The quantities make up a complex functions depending on the frequency of the wave motion. The engine speed is a variable that can be regarded as another parameter. This contribution presents a procedure how to establish the quantities coming from the suggested mathematical model and using the finite element method. In addition, formulae are derived for calculation of the terminal muffler attenuation based on the indicated characteristics. Finally, the attenuation characteristics of the terminal muffler are presented and mutually compared for several design options of the mufflers which indeed are different with respect to their geometry but have identical topology of the chambers and tailpipes.

Keywords: acoustics, attenuation, muffler geometry, transfer matrix

1. Introduction

Pollution of environment by the noise of automotive transport service raises a requirement for enhanced optimization of the whole of exhaust systems. Here the mathematical modelling becomes an indispensable tool which not only speeds up the development procedures but, in particular, it decreases the number of alternatives needed for giving effect to the prototype. The developmental acoustic computational works covering both the suction and exhaust systems including the engine as a whole are of the 1D type and include the necessity of knowing the thermo-dynamical and flow parameters of the medium. In case of transients, the solution is searched over the time domain. The harmonic analysis and the method of transfer matrices are successfully made use of in the steady running conditions. Current requirements on the precision of the calculation prediction lead to the necessity to determine the transfer matrices of the terminal mufflers using 3D models.

The specialized field of acoustics is voluminous. The Munjal's monopgraph [7] is designed to fill the need for a comprehensive resource on acoustics of mufflers and the extensive list of references can be found in it. We can also mention books [6, 2]. Simultaneously, it is possible to note that desired 3D problems are not sufficiently supported by FEM and BEM commercial software. That is, the linked string of computations, its numerical demandingness as well as capabilities to model the porous material and the perforated elements are most problematic. Modelling of acoustic waves for the flowing medium emerges to be troublesome in the case of

*Corresponding author. Tel.: +420 377 634 705, e-mail: mata@ntc.zcu.cz.

3D geometry and of the finite element method. Even further, the problems have not been sufficiently elaborated in specialized literature from the theoretical point of view and computations for the 3D geometry of mufflers are performed considering a stationary medium. Therefore, authors of the paper used an appropriate correction (see the subparagraph 5.2). Mufflers with two tailpipes have not been enough discussed in the literature of acoustics as well.

The second and fourth paragraphs of this contribution deal with the calculation methodology of the terminal muffler 3D models of the exhaust systems that contain two tailpipes. At that, it is possible to cover the walls or the perforated plate pipes as well as the porous fibre material that could fill up some muffler chamber. The formulae used for the corresponding sound transmission impedance and acoustic properties are therefore mentioned in the third paragraph. The mathematical formulation of the 3D model supposes a stationary medium. This is a feature that indeed puts the calculation work ahead as it is not necessary to establish flow field but, on the other hand, this procedure gets off partially from the real propagation of the acoustic waves in the muffler. That is why we propose a suitable and simple manipulation of the transfer matrices which eliminates the inaccuracy for the most part. Fifth paragraph presents an overview of three methods that are used preferably for the assessment of the mufflers acoustic efficiency. And finally, sixth paragraph presents and compares the results of the calculations made for the design version of terminal mufflers with two and three chambers and two tailpipes.

2. Three-dimensional waves in mufflers with a stationary medium

If the run of the engine is steady, that is its speed and power output are constant, we can consider both the flow of the combustion products and the appropriate distribution of the temperature, density and (adiabatic) sound speed unchangeable. Since the values of the Mach number are markedly less than 0.2, it is sufficient to use the methods of the linear acoustics (see e.g. [7, 6]). It can be assumed that, owing to the previously mentioned steady running condition, the transferred acoustic waves are sum of their harmonic components. The components can be expressed in terms of the acoustic pressure (scalar array) and the acoustic velocity (vector array) in the form

$$p(\boldsymbol{r})\, e^{i\omega t}\,, \quad \boldsymbol{v}(\boldsymbol{r})\, e^{i\omega t}\,, \tag{1}$$

where $\boldsymbol{r}$ denotes a position vector, t time, $\omega = 2\pi f$ is an angular frequency, and i is the imaginary unit. Generally, the quantities p and $\boldsymbol{v}$ are complex functions of the position vector $\boldsymbol{r}$ as it is necessary to keep also a watch on the harmonic waves phase shift. Then the principle of mass conservation and the law of impulse conservation give the equations

$$i\,\omega\, p + \rho\, c^2\, \nabla \cdot \boldsymbol{v} = 0\,, \quad i\,\rho\,\omega\, \boldsymbol{v} + \nabla p = 0\,, \tag{2}$$

where ρ denotes the density of the medium, c the sonic speed, ∇ is the nabla operator, ∇p is a gradient of the acoustic pressure, and $\nabla \cdot \boldsymbol{v}$ is a divergence of the acoustic velocity. It is the elimination of the acoustic velocity from (2) that gives the Helmholtz equation

$$k^2\, p(\boldsymbol{r}) + \triangle p(\boldsymbol{r}) = 0\,, \tag{3}$$

which holds for each point $\boldsymbol{r}$ in the examined region Ω. Here $k = 2\pi f/c$ means the wave number. The specification of the boundary conditions must correspond to the "topology" of the muffler which in our instance has one inlet and two outlet pipes (see Fig. 1).

The pipes are terminated by planar sections Γ_0, Γ_1, Γ_2. Let us denote the investigated acoustic quantities on a section Γ_i by p_i, $\boldsymbol{v}_i$. In the following it is possible to investigate only

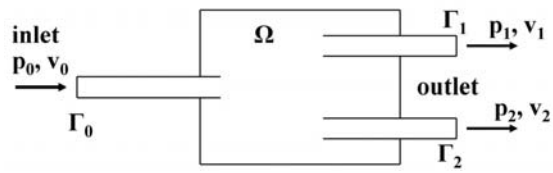

Fig. 1. Chart of the terminal muffler with one inlet and two outlet pipes

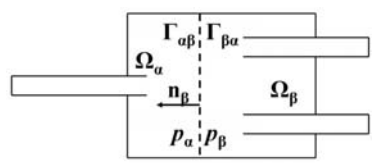

Fig. 2. Chart of the perforated wall between the muffler chambers

those acoustic waves that are planar in the pipes mentioned above, i.e. p_i, $\boldsymbol{v}_i$ are constant on Γ_i and only normal component of the vector $\boldsymbol{v}_i$ is non-zero. By analogy, let us denote it v_i. The assumption of the planeness may be seen as rightful by the fact that the waves corresponding to the higher harmonic components with non-constant profile at the section Γ_i are absorbed rapidly in the exhaust system as [7] has shown.

Second equality in (2) offers a base for the mathematical formulation of the boundary conditions. If $\boldsymbol{n}$ would denote a normal vector, $\dfrac{\partial}{\partial \boldsymbol{n}} = \boldsymbol{n} \cdot \nabla$ a derivative in the normal direction, and $v_n = \boldsymbol{n} \cdot \boldsymbol{v}$ a normal component of the acoustic velocity, we receive $\dfrac{\partial p}{\partial \boldsymbol{n}} = -i\omega\rho v_n$. For a stiff wall $\partial\Omega \backslash (\Gamma_0 \cup \Gamma_1 \cup \Gamma_2)$ we apparently have $v_n = 0$ while for anechoic outlet is $v_n = p/\rho c$ and in the general case $v_n = p/Z$ with an acoustic impedance Z.

The muffler chambers are separated by thin metal sheets that are usually perforated in their good part. Such can also be the portions of the pipes that find themselves inside the muffler. Since a detailed modelling of each of the perforation holes is unrealistic (the task would be too demanding from the computational point of view), the perforation is characterized by sound transmission impedance

$$Z_{\beta\alpha} = (p_\beta - p_\alpha)/v_{n\beta}\,, \tag{4}$$

where p_α, p_β denotes the acoustic pressure on opposed sides of the perforated walls (p_α on the surface $\Gamma_{\alpha\beta}$ and p_β on the surface $\Gamma_{\beta\alpha}$), see Fig. 2. $v_{n\beta}$ is the fictive "average" acoustic velocity through the wall area with the normal to $\partial\Omega_\beta$. Obviously, $Z_{\beta\alpha} = Z_{\alpha\beta}$ because $v_{n\beta} = -v_{n\alpha}$. With respect to (4) we obtain

$$\frac{\partial p_\beta}{\partial n_\beta} = -i\,\frac{\omega\rho}{Z}\,(p_\beta - p_\alpha) \quad \text{for} \quad \Gamma_{\beta\alpha}\,, \tag{5}$$

$$\frac{\partial p_\alpha}{\partial n_\alpha} = -i\,\frac{\omega\rho}{Z}\,(p_\alpha - p_\beta) \quad \text{for} \quad \Gamma_{\alpha\beta}\,. \tag{6}$$

3. Acoustic impedance

The value of the sound transmission impedance Z appearing in the relation (4) was an object of the research and measuring at a number of acoustic workplaces. One of the very often used

relations presented by Sullivan et al. [10]. Kirby and Cummings [4] extended it to cover even the situation when the sheet is surrounded by an absorbing porous material and have proposed the formula

$$Z = \rho\, c \left[0,006 + ik \left\{ t_w + 0,375\, d_h \left(1 + \frac{\tilde{\rho}\tilde{c}}{\rho c} \frac{\tilde{k}}{k} \right) \right\} \right] / \Phi \,. \tag{7}$$

The symbol t_w denotes the wall thickness, d_h diameter of the holes and Φ is a porosity of the sheet. Further the complex values of the characteristic impedance $\tilde{\rho}\tilde{c}$ and wave number $\tilde{k}$ of the absorbing material were used. Since the structure of the materials is complicated, their acoustic properties are being preferably determined in an experimental way. With this fact in view, we use the relations from the Huff's work [3] that comes out from the measurements carried out in the Owens Corming Laboratories made for the automobile mufflers absorbing materials

$$\tilde{\rho}\tilde{c}/\rho c \;=\; \left[1 + 0,085\,5\,(f/R)^{-0,754}\right] + i\left[-0,076\,5\,(f/R)^{-0,732}\right] \,, \tag{8}$$

$$\tilde{k}/k \;=\; \left[1 + 0,1472\,(f/R)^{-0,577}\right] + i\left[-0,173\,4\,(f/R)^{-0,595}\right] \,. \tag{9}$$

The quantities $\tilde{\rho}\,\tilde{c}$ and $\tilde{k}$ are consequently dependent on the acoustic waves frequency f when R denotes the flow resistivity of the filling material.

4. Finite element method discretization

We obtain the weak solution of the Helmholtz equation (3) by multiplying it by the test function ξ and by applying the first Green formula $\int_\Omega \xi \,\triangle\, p\, dv = \int_{\partial\Omega} \xi \,\dfrac{\partial p}{\partial n}\, \mathrm{d}S - \displaystyle\int_\Omega \nabla\xi \cdot \nabla p\, \mathrm{d}r$. Next we replace the normal derivative $\dfrac{\partial p}{\partial n}$ by the expressions for the specified boundary conditions that were derived in the second article. If the anechoic output is the case, we obtain the relation

$$\begin{aligned} \int_\Omega \left(\widehat{k}^2 \xi\, p - \nabla\xi \cdot \nabla p \right) \mathrm{d}r - i\omega \int_{\Gamma_1} \frac{1}{c}\, \xi\, p\, \mathrm{d}S - i\omega \int_{\Gamma_2} \frac{1}{c}\, \xi\, p\, \mathrm{d}S + i\omega \int_{\Gamma_{\beta\alpha}} \frac{\widehat{\rho}}{Z}\, \xi_\beta \left(p_\beta - p_\alpha\right) \mathrm{d}S \\ - i\omega \int_{\Gamma_{\alpha\beta}} \frac{\widehat{\rho}}{Z}\, \xi_\alpha \left(p_\alpha - p_\beta\right) \mathrm{d}S = i\omega\rho \int_{\Gamma_0} \xi\, v_{0n}\, \mathrm{d}S \end{aligned} \tag{10}$$

at the sections Γ_1 a Γ_2, where ξ_α, p_α resp. ξ_β, p_β denote traces of the functions ξ and p on the boundary $\partial\Omega_\alpha$ resp. $\partial\Omega_\beta$ of the corresponding regions. $\widehat{k}$ replaces k or $\tilde{k}$ depending on whether the position vector $\boldsymbol{r}$ denotes a point of the subregion $\Omega\backslash\tilde{\Omega}$ without absorption material or of the subregion $\tilde{\Omega}$ with absorption material; $\widehat{\rho}$ has a similar meaning.

To discretize the equation (10), let us consider the functions $\xi(\boldsymbol{r})$ and $p(\boldsymbol{r})$ as linear combinations of shape functions $N_j(\boldsymbol{r})$ involved in the finite element method

$$p\left(\boldsymbol{r}\right) = \sum_j p_j N_j\left(\boldsymbol{r}\right) \,, \quad \xi\left(\boldsymbol{r}\right) = \sum_j \xi_j N_j(\boldsymbol{r}), \tag{11}$$

where we write $p_j = p\left(\boldsymbol{r}_j\right)$, $\xi_j = \xi\left(\boldsymbol{r}_j\right)$ for the j-th node of the discretization with a position vector $\boldsymbol{r}_j$. We then receive a system of linear algebraic equations

$$\left[-\mathbf{K} - ik\,\mathbf{B}_1 - ik\,\mathbf{B}_2 + i\frac{\widehat{\rho}\widehat{c}}{Z}\,\widehat{k}\,\mathbf{D}_{\beta\alpha} + i\frac{\rho c}{Z}\,k\,\mathbf{D}_{\alpha\beta} + k^2\,\mathbf{M}_{\Omega-\widetilde{\Omega}} + \widetilde{k}^2\,\mathbf{M}_{\widetilde{\Omega}} \right] \boldsymbol{p} = i\rho c k\,\mathbf{B}_0 \mathcal{V}_{0n} \,. \tag{12}$$

In doing so, we supposed that the quantities $\widehat{\rho}$, $\widehat{c}$, $\widehat{k}$ are constant over the subregions $\tilde{\Omega}$ and $\Omega - \tilde{\Omega}$ which is a satisfactory approximation of the situation in the terminal mufflers. This simplification however saves a lot of computational time when setting up the stiffness matrix. Among the presented matrices the matrices $\mathbf{K}$, $\mathbf{B}_1$, $\mathbf{B}_2$, $\mathbf{M}_{\Omega-\widetilde{\Omega}}$ and $\mathbf{M}_{\widetilde{\Omega}}$ are symmetric with

$$
\begin{aligned}
\mathbf{K} &= [k_{ij}] \quad \text{with} \quad k_{ij} = \int_{\Omega} \nabla N_i(\boldsymbol{r}) \cdot \nabla N_j(\boldsymbol{r})\, \mathrm{d}\boldsymbol{r}\,, \\
\mathbf{B}_1 &= [b_{ij}] \quad \text{with} \quad b_{ij} = \int_{\Gamma_1} N_i(\boldsymbol{r})\, N_j(\boldsymbol{r})\, \mathrm{d}S\,, \\
\mathbf{B}_2 &= [b_{ij}] \quad \text{with} \quad b_{ij} = \int_{\Gamma_2} N_i(\boldsymbol{r}) N_j(\boldsymbol{r})\, \mathrm{d}S\,, \\
\mathbf{B}_0 &= [b_{ij}] \quad \text{with} \quad b_{ij} = \int_{\Gamma_0} N_i(\boldsymbol{r}) N_j(\boldsymbol{r})\, \mathrm{d}S\,, \\
\mathbf{M}_* &= [m_{ij}] \quad \text{with} \quad m_{ij} = \int_{*} N_i(\boldsymbol{r})\, N_j(\boldsymbol{r})\, \mathrm{d}r\,, \quad * \equiv \Omega - \widetilde{\Omega},\, \widetilde{\Omega}\,.
\end{aligned}
\tag{13}
$$

Next $\boldsymbol{p} = (\dots, p_j, \dots)^T$ is a vector of unknown values of acoustic pressure at the mesh nodes $\boldsymbol{r}_j$ and $\mathcal{V}_{0n} = (\dots, v_{0n_i}, \dots)^T$ with $v_{0n_i} = v_{0n}$ for a node i fulfilling $\boldsymbol{r}_i \in \Gamma_0$ or otherwise $v_{0n_i} = 0$. It holds

$$
\mathbf{D}_{\beta\alpha} = \left[(-1)^{f(\beta,*)} d_{i\beta,\,j*}\right]\,, \quad \mathbf{D}_{\alpha\beta} = \left[(-1)^{f(\alpha,*)} d_{i\alpha,\,j*}\right]\,, \tag{14}
$$

for the matrices representing the sound transmission impedance of the performed walls where

$$
* = \alpha, \beta \quad \text{and} \quad f(\beta,*) = \begin{cases} 1 & \text{for } * = \beta\,, \\ 0 & \text{for } * = \alpha\,, \end{cases} \quad f(\alpha,*) = \begin{cases} 0 & \text{for } * = \beta\,, \\ 1 & \text{for } * = \alpha\,, \end{cases} \tag{15}
$$

with the position vector $\boldsymbol{r}_i$ being processed with the pair of conjugate nodes i_α, i_β of the finite element mesh when the first of which lies on the surface $\Gamma_{\alpha\beta}$ and the other on $\Gamma_{\beta\alpha}$. Finally $d_{i*,\,j\#} = \int_{\Gamma_{*\#}} N_i(\boldsymbol{r}) N_j(\boldsymbol{r}) dS$ and the sum $\mathbf{D}_{\beta\alpha} + \mathbf{D}_{\alpha\beta}$ creates a symmetric matrix. Consequently, the matrix of the system (12) is symmetric for a muffler without absorption material.

A way of verifying the implementation of the mathematical model and numerical discretization is described in [5]. It may be stated that the agreement of the calculation results with the known analytic relations and also with the experimental results (see e.g. SAE [9]) is excellent in case of motionless medium.

5. The exhaust muffler as an acoustic filter

5.1. The transfer matrix

It is useful to represent the exhaust system by analogy with electric circuit by means of the acoustic filters theory (see e.g. [6, 7]). The terminal muffler is then one of the acoustic elements of such "circuit". This element is described mathematically by a so called transfer matrix which is of the order 2x4 in case of two tailpipes. Consequently it is defined by eight complex numbers A, B, C, D, E, F, G, H (let us throw in a remark that this is true for specified engine revolution speed and selected wave frequency). Following the notation of the Fig. 1, the relation between

the acoustic quantities p_0, v_0 on input and the quantities p_1, v_1, p_2, v_2 on output is as follows

$$\begin{bmatrix} p_0 \\ v_0 \end{bmatrix} = \begin{bmatrix} A & B & C & D \\ E & F & G & H \end{bmatrix} \begin{bmatrix} p_1 \\ v_1 \\ p_2 \\ v_2 \end{bmatrix}. \tag{16}$$

In case of anechoic conditions $v_1 = p_1/\rho c$, $v_2 = p_2/\rho c$ in the tailpipes and with the choice $v_0 = 1$, we can rewrite the relation (16) to the form

$$\begin{bmatrix} {}^1p_0 \\ 1 \end{bmatrix} = \begin{bmatrix} A & B & C & D \\ E & F & G & H \end{bmatrix} \begin{bmatrix} {}^1p_1 \\ {}^1p_1/\rho c \\ {}^1p_2 \\ {}^1p_2/\rho c \end{bmatrix}. \tag{17}$$

Having solved the task (12) we obtain the values of acoustic pressures ${}^1p_0$, ${}^1p_1$, ${}^1p_2$. It is useful to remain aware of the fact that all that is necessary is only single choice of the input value v_0 of the sound speed. The relation (17) is a linear system of two equations for eight unknowns $A, B, \ldots, H$. It is therefore necessary to get additional information for setting up the transfer matrix. We can partly insert stiff impermeable walls (then $v_i = 0$) at the sections Γ_0, Γ_1, Γ_2 or we can prescribe even non-zero values of the sound speed at these sections. We can again formulate the associated problems of establishing the acoustic pressures at the sections Γ_0, Γ_1, Γ_2 in the form (12) where $\mathbf{B}_1$, $\mathbf{B}_2$ can be zero or the matrix $\mathbf{B}_0$ can belong to some different section Γ_i. It involves only a simple modification of the input data, not a modification of the program code solver. We can therefore add

$$\begin{gathered} \begin{bmatrix} {}^2p_0 \\ 1 \end{bmatrix} = \begin{bmatrix} A & B & C & D \\ E & F & G & H \end{bmatrix} \begin{bmatrix} {}^2p_1 \\ 0 \\ {}^2p_2 \\ 0 \end{bmatrix}, \quad \begin{bmatrix} {}^3p_0 \\ 1 \end{bmatrix} = \begin{bmatrix} A & B & C & D \\ E & F & G & H \end{bmatrix} \begin{bmatrix} {}^3p_1 \\ {}^3p_1/c\rho \\ {}^3p_2 \\ 0 \end{bmatrix}, \\ \begin{bmatrix} {}^4p_0 \\ 0 \end{bmatrix} = \begin{bmatrix} A & B & C & D \\ E & F & G & H \end{bmatrix} \begin{bmatrix} {}^4p_1 \\ 0 \\ {}^4p_2 \\ -1 \end{bmatrix} \end{gathered} \tag{18}$$

to (17). Of here, a direct way leads to the equation systems

$$\begin{bmatrix} {}^1p_1 & {}^1p_1/\rho c & {}^1p_2 & {}^1p_2/\rho c \\ {}^2p_1 & 0 & {}^2p_2 & 0 \\ {}^3p_1 & {}^3p_1/\rho c & {}^3p_2 & 0 \\ {}^4p_1 & 0 & {}^4p_2 & -1 \end{bmatrix} \begin{bmatrix} A \\ B \\ C \\ D \end{bmatrix} = \begin{bmatrix} {}^1p_0 \\ {}^2p_0 \\ {}^3p_0 \\ {}^4p_0 \end{bmatrix}, \tag{19}$$

$$\begin{bmatrix} {}^1p_1 & {}^1p_1/\rho c & {}^1p_2 & {}^1p_2/\rho c \\ {}^2p_1 & 0 & {}^2p_2 & 0 \\ {}^3p_1 & {}^3p_1/\rho c & {}^3p_2 & 0 \\ {}^4p_1 & 0 & {}^4p_2 & -1 \end{bmatrix} \begin{bmatrix} E \\ F \\ G \\ H \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \end{bmatrix}, \tag{20}$$

the solution of which provides all terms $A, B, \ldots, H$ of the transfer matrix searched after.

5.2. Waves in a circular duct with moving medium and the modification of the transfer matrix

Let us consider a pipe of circular section the length of which is L and the moving medium is characterized by the Mach number M. The flowing of the medium may be assumed to be inviscid for our purposes owing to the diameters of the terminal mufflers pipes. The planar harmonic wave at a distance of x from the beginning of the pipe is described by the relations (for a theoretical background, see for example [5,6])

$$\begin{aligned} p(x)\, e^{i\omega t} &= \left(p_+\, e^{-\frac{ik}{1+M}x} + p_-\, e^{\frac{ik}{1-M}x}\right) e^{i\omega t}\,, \\ v(x)\, e^{i\omega t} &= \frac{1}{\rho c}\left(p_+\, e^{-\frac{ik}{1+M}x} - p_-\, e^{\frac{ik}{1-M}x}\right) e^{i\omega t}\,. \end{aligned} \tag{21}$$

First term containing p_+ represents an input incident wave while the other term containing p_- describes a reflection wave. The transfer matrix 2x2 of the pipe under consideration with the length of L and Mach number M is then (see e.g. [5,6])

$$\mathbf{Q}(L,M) = \begin{bmatrix} e^{\frac{ik}{1+M}L} + e^{-\frac{ik}{1-M}L} & \rho c\left(e^{\frac{ik}{1+M}L} - e^{-\frac{ik}{1-M}L}\right) \\ \dfrac{1}{\rho c}\left(e^{\frac{ik}{1+M}L} - e^{-\frac{ik}{1-M}L}\right) & e^{\frac{ik}{1+M}L} + e^{-\frac{ik}{1-M}L} \end{bmatrix}. \tag{22}$$

The sections of separate terminal mufflers chambers are in size several fold greater than the input and output pipes. On the contrary, the Mach numbers of the chambers are substantially less than is the case with the pipes. We can therefore conclude that the influence of the flow of the medium through the chambers on the change of acoustic waves transfer is much less than the influence of the flow in the muffler pipe contrary to the case of motionless medium. For this reason, for the flowing medium, we modify the transfer matrix appearing in the relation (16) as follows

$$\begin{bmatrix} A & B & C & D \\ E & F & G & H \end{bmatrix} := \\ \mathbf{Q}(M_0,L_0)\mathbf{Q}^{-1}(0,L_0) \begin{bmatrix} A & B & C & D \\ E & F & G & H \end{bmatrix} \begin{bmatrix} \mathbf{Q}^{-1}(0,L_1)\mathbf{Q}(M_1,L_1) & 0 \\ 0 & \mathbf{Q}^{-1}(0,L_2)\mathbf{Q}(M_2,L_2) \end{bmatrix}.$$

5.3. Muffler performance parameters

If the coordinate has the value $x = 0$, the acoustic pressure and acoustic speed of the wave motion are given by the formulae

$$p(0) = p_+ + p_-, \qquad v(0) = (p_+ - p_-)/\rho c\,. \tag{23}$$

Acoustic power output transferred by the pipe is (see e.g. [6])

$$W(M) = \frac{1}{2}\frac{S|p_+|^2}{\rho c}\left((1+M)^2 - |R|^2\,(1-M)^2\right), \tag{24}$$

where $R = p_-/p_+$ is a reflection factor, S is the size of the pipe section. The input power transferred by the actual incidental wave is then

$$W_{in}(M) = \frac{1}{2}\frac{S|p_+|^2}{\rho c}\,(1+M)^2\,. \tag{25}$$

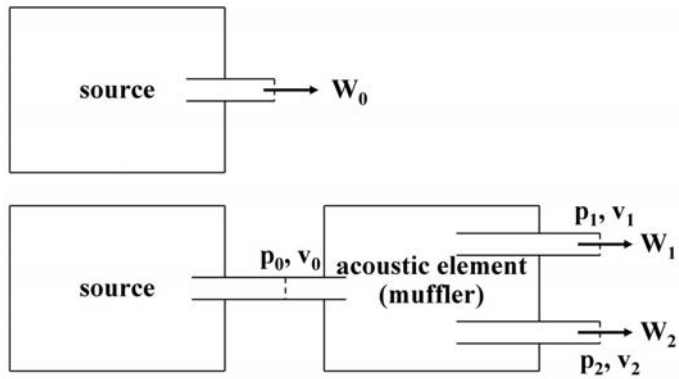

Fig. 3. Chart of an acoustic system with a source without a terminal muffler and a system including it

As a rule, the efficiency of the acoustic mufflers is evaluated by one of the following parameters:

– Insertion loss, IL;
– Transmission loss, TL;
– Level difference, LD.

First of the parameters, the insertion loss, is defined as a difference between the level of the radiated acoustic power of a system with the evaluated acoustic e lement missing (terminal muffler in our instance) and a system including it (see Fig. 3)

$$IL = L_{W_0} - L_{W_1+W_2} = 10\,\log \frac{W_0}{W_1 + W_2} \qquad [\mathrm{dB}]\,. \tag{26}$$

The corresponding radiation impedance Z_R, i.e. $p_i = v_i Z_R$, $i = 1, 2$ (see e.g. [7, 6]) can be determined up to the termination of the tailpipes. The knowledge of the transfer matrix of a muffler as an acoustic element certainly does not enable to determine IL because we don't know the impedance of the source. Different design options of the mufflers can nevertheless be mutually compared provided that the acoustic pressure of the source is constant. It is not necessary to have knowledge of the source if the transmission loss TL should be determined. This parameter is defined as a difference between the acoustic power level of the incident wave and the level of the output power at the anechoic termination of the pipes ($p_i = \rho c\, v_i$, $i = 1, 2$)

$$TL = L_{W_{in}} - L_{W_1+W_2} = 10\,\log \frac{W_{in}}{W_1 + W_2} \qquad [\mathrm{dB}]\,. \tag{27}$$

Denoting $q = p_1/p_2$ we derive

$$TL = 10\,\log \frac{1}{4}\frac{S_0}{S}\frac{\rho c}{\rho_0 c_0}\frac{|\,Aq + Bq/\rho c + C + D/\rho c + Eq\,\rho_0 c_0 + Fq\,\rho_0 c_0/\rho c\,|^2}{(1+M_1)^2\,|q|^2 + (1+M_2)^2} \tag{28}$$

using (16), (23), (24), (25) (26) and an anechoic condition. Here we assume that both output pipes have equal cross sections S as well as the medium impedances ρc. S_0 resp. $\rho_0 c_0$ denotes the cross section size of the input pipe resp. the impedance of the medium at the input. Parameter TL however cannot give fully satisfactory answer to the question about the muffler behaviour in a real exhaust system given a specified engine and other aggregates. It is in line with the non-necessity to use the formula (28), that is all elements of the transfer matrix, for determining TL. It leads to four-fold solution of the system of the form (12) for the corresponding

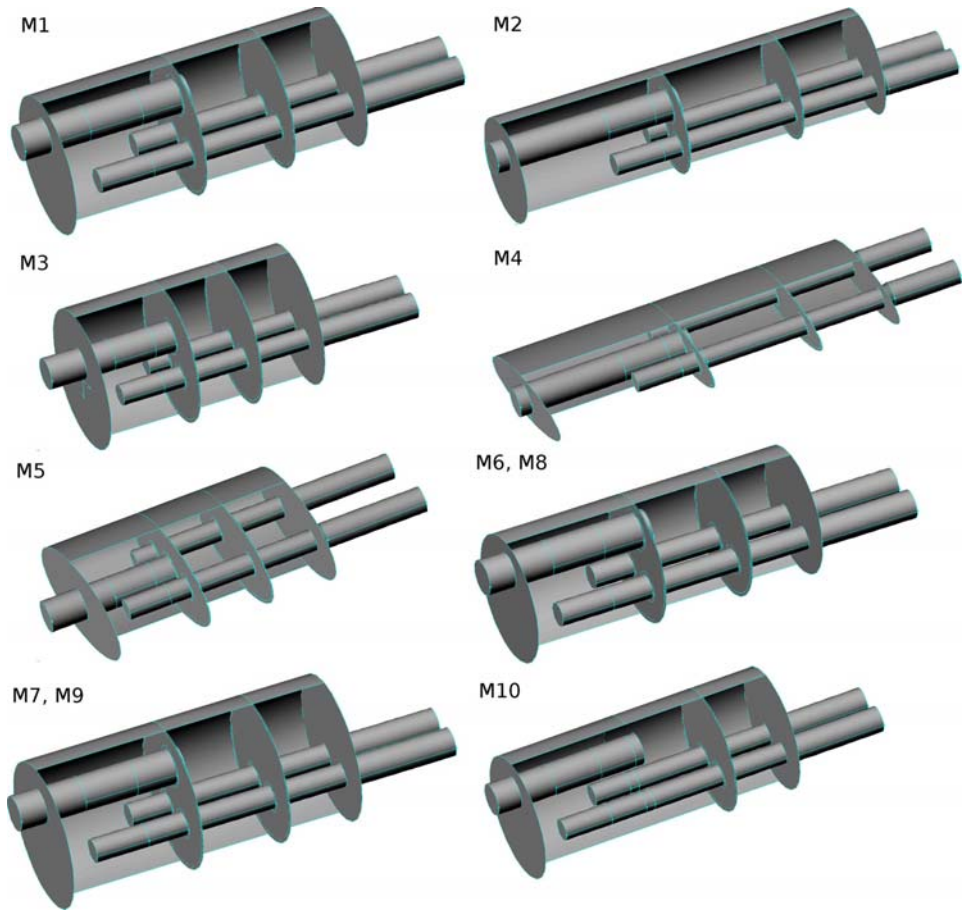

Fig. 4. Terminal mufflers

problems (17), (18), (19) and (20). Considering TL, the system (12) can be solved only once and then we use the values ${}^1p_0$, ${}^1p_1$, ${}^1p_2$ of the problem (17) of the acoustic pressure in another suitably selected point of the input pipe (analogy of the so called 3-pole method [5]).

The third parameter for the evaluation of the acoustic mufflers efficiency is the level difference, LD. It is the difference of acoustic pressure levels at selected sections of the inlet and outlet pipe

$$LD = 20 \log |\frac{p_0}{p_1}| \qquad [\mathrm{dB}]\,. \tag{29}$$

But accurate comparison between two mufflers by the parameter LD requires the knowledge of the source impedance.

6. Comparison of selected design options of the mufflers

The presented computational methodology and evaluation of the efficiency were applied in the problems of terminal mufflers comprising two or three chambers and two tailpipes as is shown in the Fig. 4.

Table 1. Inherent parameters of mufflers in view

Muffler	Capacity [dm^3]	Lengths of chambers [mm]	Diameter [mm]	Third chamber
M1	15.4	200/135/110	206	porous material
M2	15.4	280/195/140	176	porous material
M3	15.4	155/108/72	236	porous material
M4	15.4	280/195/140	360/86	porous material
M5	15.4	155/108/72	373/105	porous material
M6	13.4	200/135/110	192	porous material
M7	17.5	200/135/110	220	porous material
M8	13.4	200/135/110	192	exhaust gas
M9	17.5	200/135/110	220	exhaust gas
M10	15.4	2 chambers, 335/110	206	porous material

Base parameters of mufflers under consideraton are given in the Tab. 1. The chambers are separated by perforated sheets with the thickness $t_w = 1.2$ mm, diameter $d_h = 3.5$ mm and the porosity $\Phi = 0.26$. Curves with their transmission loss TL in the frequency interval from 30 Hz to 400 Hz are depicted in the Fig. 5. Parameters of flowing medium correspond to a car four-cylinder engine under the extremely low revolutions 1 000 rpm. The acoustic attenuation for the one-octave band 63 Hz is in our interest above all (see Fig. 6). The reason is that car factories have some troubles with this octave band in the case of four-cylinder engines. We can see that mufflers with porous material in the third chamber are better than the same mufflers without it. Further, the muffler capacity and lengths of chambers are crucial parameters as far as the transmission loss is concerned.

7. Conclusion

The work deals with the calculation methodology of the terminal muffler 3D models of the exhaust systems that contain two tailpipes. Computational procedure based on FEM is proposed with the aim to be of utmost low-cost. A suitable modification of the computed transfer matrix is then suggested because FEM calculations are carried out for the steady medium. The modificated transfer matrix is wanted description of the muffler as an acoustic filter.

To illustrate the calculation methodology, the transmission loss versus frequency are shown in Fig. 5 and 6 for several mufflers on an car four-cylinder engine operating at 1 000 prm. We can see obvious impact of the muffler capacity (see the muffler M7) and of the chambers lengths on an acoustic attenuation (see the mufflers M2, M4) for the one-octave band 63 Hz which is first of all in our interest. We can also see the impact of the tailpipes length and diameter. Reducing the diameter is the cause of increasing the Mach number of the flowing medium and consequently the acoustic attenuation.

Comparing pairs of mufflers M8–M6 and M9–M7 we can quantify the positive influence of porous material filling their third chambers. M2 and M4 are the best ones if the front muffler (not considered in the paper) of an exhaust system damps down amplitudes of acoustic waves in the interval of frequences from 300 Hz.

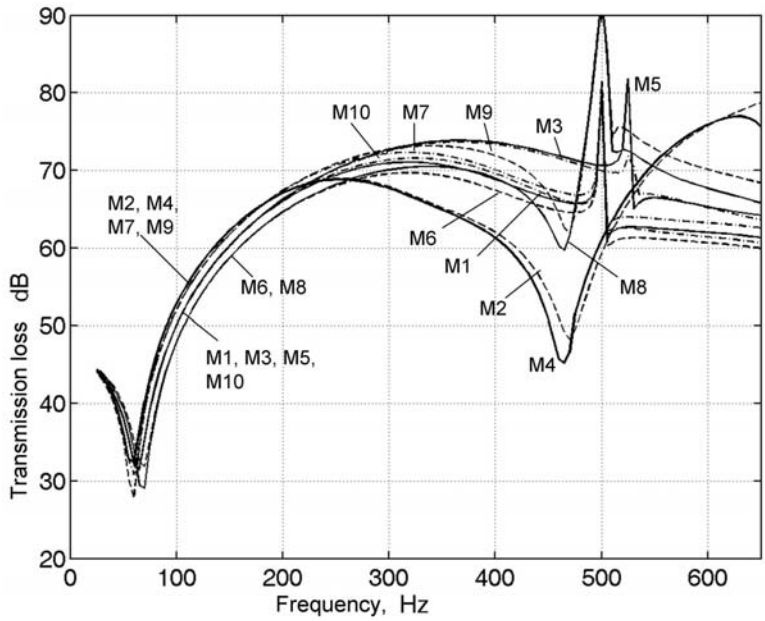

Fig. 5. Transmission loss versus frequency for the mufflers M1–M10 from tab. 1

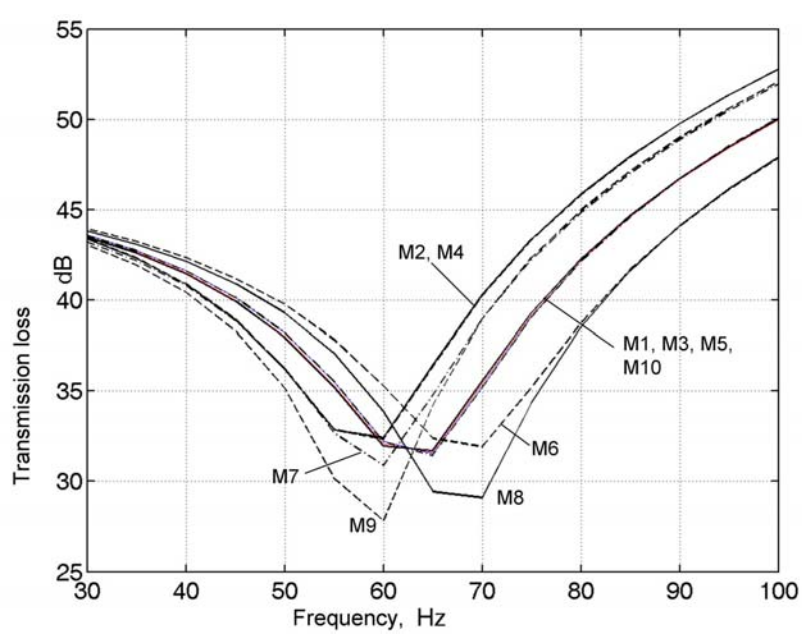

Fig. 6. Transmission loss versus frequency for the mufflers M1–M10 from tab. 1

Acknowledgements

The paper is based on work supported by the Czech Ministry of Education under the research project 1M06031.

References

[1] M. E. Delany, E. N. Bazley, Acoustical properties of fibrous absorbent materials, Applied Acoustics 3(1970), pp. 105–116.

[2] Handbook of Noise and Vibration Control, edited by Crocker, M. J., John Wiley & Sons, 2007.

[3] N. T. Huff, Materials for absoptive silencer systems, SAE Technical Paper Series, 2001-01-1458. Reprinted from: Proceedings of the 2001 Noise and Vibration Conference, Traverse City, Michigan.

[4] R. Kirby, A. Cummings, The impendance of perforated plates subjected to grazing gas flow and backed by porous media, J. Sound and Vibration 217(4)(1998), pp. 619–636.

[5] R. Matas, J. Voldřich, B. Tikal, A finite element method for acoustic attenuation performance in mufflers, Proceedings of the 22nd conference Computational Mechanics 2006, Nečtiny, 2006. (in Czech)

[6] F. P. Mechel, Formulas of Acoustics, Springer, Berlin, Heildelberg, New York, 2002.

[7] M. L. Munjal, Acoustics of Ducts and Mufflers, John Wiley & Sons, New York, Chichester, Brisbane, Toronto, Singapore, 1987.

[8] A. Selamet, I. J. Lee, Z. L., Ji, N. T. Huff, Acoustic attenuation performance of perforated absorbing silencers, SAE Technical Paper Series, 2001-01-1435. Reprinted from: Proceedings of the 2001 Noise and Vibration Conference, Traverse City, Michigan.

[9] A. Selamet, M. B. Xu, I.-L. Lee, Analytical approach for sound attenuation in perforated dissipative silencers, J. Acoust. Soc. Am. 115(2004), pp. 2 091–2 099.

[10] J. W. Sullivan, M. J. Crocker, Analysis of concentric-tube resonators having unpartitioned cavities, J. Acoust. Soc. Am. 64(1978), pp. 207–215.

[11] T. W. Wu, G. C. Wan, Muffler performance studies using direct mixed-body boundary element method and a three-point method for evaluating transmission loss, J. Vib. Acoust. 118(1996), pp. 479–484.