UNIVERSITY OF WEST BOHEMIA

Applied and Computational Mechanics 3 (2009) 39–50

Applied and Computational Mechanics

Optimization of the railway vehicle bogie in term of dynamics

Z. Hlaváč[a,*], V. Zeman[a]

[a] *Faculty of Applied Sciences, University of West Bohemia, Univerzitní 22, 306 14 Plzeň, Czech Republic*

Received 25 August 2008; received in revised form 27 February 2009

Abstract

The paper deals with parameter optimization method of a railway vehicle bogie. Wheelsets are subjected to stochastic excitation caused by irregularities of the track geometry and/or caused by deterministic excitation from polygonalized running surface of the wheels, respectively. Stochastic irregularities are understood as stationary stochastic process described by spatial power spectral density functions. The polygonalization process forms radial irregularities of the running surface which can be described by of a harmonic function depending on order of the wheel polygonalization. The optimization method is based on minimization of several types of objective functions suitable for optimization from the dynamic load point of view of exposed linkage.

Keywords: optimization, railway vehicle bogie, dynamic loading, kinematic excitation, track irregularities, radial wheel irregularities

1. Introduction

Modern high-speed railway vehicle show some dynamic phenomena characterized by frequencies in the mid-frequency range. To described these phenomena conventional models on the basis of rigid multibody-systems [5] are not sufficient.

Detailed models of railway vehicle components were presented for example in the paper [3] and there cited papers. None of mentioned works contains complex and detailed models of railway vehicle bogie respecting spatial vibrations of all bogie components and visco-elastic couplings among them e.g. gearing, clutches, supports of engine stators and of gear housings to bogie frame, elastic wheelset axles and flexible wheels etc.

This article presents the complex mathematical model of the railway vehicle bogie with two individual wheelset drives (Fig. 1) of the electric locomotive developed for speeds about 200 km/h by the company ŠKODA TRANSPORTATION s.r.o. The model respects spatial vibrations of all drive components as a consequence of soft supports of engine stators and of drive housings of both individual drives (ID1, ID2) to bogie frame (BF) by rubber silent blocks. Their centres of elasticity are designated A_1, B_1, C_1 (for ID1) and A_2, B_2, C_2 (for ID2). An elasticity of composite hollow shafts embracing the wheelset axles as well as the elasticity of wheelsets moving on visco-elastic railway balast are taken into account.

The main aim of this article is to present an original method of the optimization of the chosen design bogie parameters from the dynamic load point of view of exposed linkage expressed by vertical dynamic forces acting between rails and wheels and by dynamic forces transmitted by rubber silent blocks. Excitations by track irregularities [7] and polygonalized running surfaces of the wheels [3] are considered.

*Corresponding author. Tel.: +420 377 632 331, e-mail: hlavac@kme.zcu.cz.

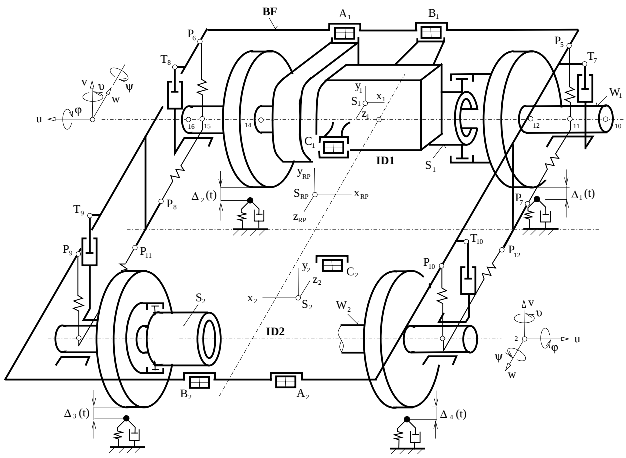

Fig. 1. Scheme of the boogie

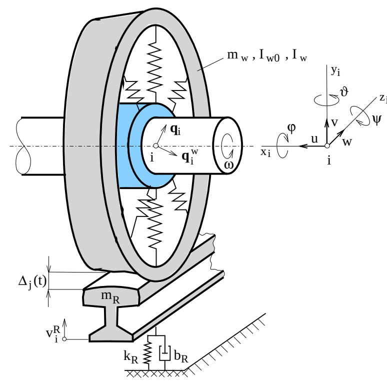

Fig. 2. Scheme of the elastic wheel

2. Mathematical model of the bogie

The development of the linearized mathematical model of the bogie (Fig. 1) with rigid wheels was written in the contribution [9] and detailed in the research report [10]. Elastic wheels reduce the unsprung mass and isolate the bogie frame from the excitation caused by the wheel and rail interaction. The radial and lateral elastic wheels, specially with a composite material, consists of very stiff parts like the rim and the disc, and a relatively soft connection between both parts. Therefore the wheel rims and the wheel discs can be considered as rigid bodies [3]. The flexible connection in between can be represented in coordinate system x_i, y_i, z_i by linear massless springs (see Fig.2) characterized by radial stiffness $k_y^w = k_z^w$, lateral stiffness k_x^w, torsional stiffness k_{xx}^w and bending stiffness $k_{yy}^w = k_{zz}^w$. Each wheel rim may undergo lateral, vertical, longitudinal, torsional, yaw and roll motion written by the displacement vector

$$\mathbf{q}_i^w = [u_i^w, v_i^w, w_i^w, \varphi_i^w, \vartheta_i^w, \psi_i^w]^T,\ i = 12, 14 \tag{1}$$

for ID1 and ID2, where the subscript i corresponds to nodal point $(i = 12, 14)$ at the wheelset axis, to which is fixed wheel on the axis. Displacements of corresponding wheel discs are expressed by the vector

$$\mathbf{q}_i = [u_i, v_i, w_i, \varphi_i, \vartheta_i, \psi_i]^T,\ i = 12, 14. \tag{2}$$

The mathematical model of the bogie with rigid wheels [9] was derived in configuration space

$$\mathbf{q} = [\mathbf{q}_{ID1}^T, \mathbf{q}_{BFCB}^T, \mathbf{q}_{ID2}^T]^T \tag{3}$$

of dimension 165, where subvectors correspond to three subsystems- individual drives (ID1 and ID2) that include couplings among drive components (gearing, disc clutch, composite hollow shaft, claw clutch, railway balast) and the bogie frame linked by secondary suspension and dampers with a half of car body (BFCB). The displacement vectors of the individual drives with elastic wheels are extended into the form

$$\tilde{\mathbf{q}}_{ID1} = \begin{bmatrix} \mathbf{q}_{ID1} \\ \bar{\mathbf{q}}_{12}^w \\ \bar{\mathbf{q}}_{14}^w \end{bmatrix}, \quad \tilde{\mathbf{q}}_{ID2} = \begin{bmatrix} \mathbf{q}_{ID2} \\ \bar{\bar{\mathbf{q}}}_{12}^w \\ \bar{\bar{\mathbf{q}}}_{14}^w \end{bmatrix}, \tag{4}$$

where vectors of node displacements with one bar correspond to wheel rims of ID1 and with two bars to wheel rims of ID2, respectively. A contribution of the wheel rims to mass and stiffness matrix of the subsystem ID1 (similarly for ID2) can be derived from Lagrange's equations on the basis of their kinetic E_k^w and potential E_p^w energy including influence of track structure

$$E_k^w = \frac{1}{2}(\dot{\bar{\mathbf{q}}}_{12}^w)^T \mathbf{M}_w \dot{\bar{\mathbf{q}}}_{12}^w + \frac{1}{2}(\dot{\bar{\mathbf{q}}}_{14}^w)^T \mathbf{M}_w \dot{\bar{\mathbf{q}}}_{14}^w + \frac{1}{2} m_R (\dot{v}_{12}^w - \dot{\Delta}_1)^2 + \frac{1}{2} m_R (\dot{v}_{14}^w - \dot{\Delta}_2)^2 , \quad (5)$$

$$E_p^w = \frac{1}{2}(\bar{\mathbf{q}}_{12}^w - \mathbf{q}_{12})^T \mathbf{K}_w (\bar{\mathbf{q}}_{12}^w - \mathbf{q}_{12}) + \frac{1}{2}(\bar{\mathbf{q}}_{14}^w - \mathbf{q}_{14})^T \mathbf{K}_w (\bar{\mathbf{q}}_{14}^w - \mathbf{q}_{14}) + + \frac{1}{2} k_R (v_{12}^w - \Delta_1)^2 + \frac{1}{2} k_R (v_{14}^w - \Delta_2)^2 , \quad (6)$$

where the mass and stiffness matrices of the wheel rim is

$$\mathbf{M}_w = \text{diag}\,(m_w, m_w, m_w, I_{w0}, I_w, I_w),\; \mathbf{K}_w = \text{diag}\,(k_x^w, k_y^w, k_z^w, k_{xx}^w, k_{yy}^w, k_{zz}^w)$$

and damping matrix $\mathbf{B}_w$ has the same structure as the stiffness matrix $\mathbf{K}_w$. The whole track structure (rail, railpad, sleeper and balast) is reduced to a single mass-spring-damper system [4] defined by mass, stiffness and damping parameters m_R, k_R, b_R. The wheelset kinematic excitation by vertical track irregularities and/or polygonalized running surface of the wheel rims is expressed by deviations Δ_j (for ID1 $j = 1, 2$ and for ID2 $j = 3, 4$ — see Fig. 1).

In the configuration space defined by the vector of generalized coordinates

$$\mathbf{q} = [\tilde{\mathbf{q}}_{ID1}^T, \mathbf{q}_{BFCB}^T, \tilde{\mathbf{q}}_{ID2}^T]^T \quad (7)$$

of dimension 189, the mathematical model of the bogie has the form (detailed in [9], [10])

$$\mathbf{M}\ddot{\boldsymbol{q}}(t) + \mathbf{B}\dot{\boldsymbol{q}}(t) + \mathbf{K}\boldsymbol{q}(t) = \mathbf{f}_G + \mathbf{f}_E(\dot{\boldsymbol{q}}) + \mathbf{f}_{R,W}(\mathbf{q}, \dot{\boldsymbol{q}}, t) , \quad (8)$$

where matrices have the block-diagonal structure

$$\mathbf{M} = \text{diag}\,(\tilde{\mathbf{M}}_{ID}, \mathbf{M}_{BFCB}, \tilde{\mathbf{M}}_{ID}) ,$$

$$\mathbf{B} = \text{diag}\,(\tilde{\mathbf{B}}_{ID}, \mathbf{B}_{BFCB}, \tilde{\mathbf{B}}_{ID}) + \mathbf{B}_{D,BF} + \mathbf{B}_{W,BF} , \quad (9)$$

$$\mathbf{K} = \text{diag}\,(\tilde{\mathbf{K}}_{ID}, \mathbf{K}_{BFCB}, \tilde{\mathbf{K}}_{ID}) + \mathbf{K}_{D,BF} + \mathbf{K}_{W,BF}$$

corresponding to subsystems. With respect to matrices of the bogie with rigid wheels [9], mass, damping and stiffness matrices $\tilde{\mathbf{M}}_{ID}, \tilde{\mathbf{B}}_{ID}, \tilde{\mathbf{K}}_{ID}$, of the individual drives without linkages at the bogie frame are extended by 24 DOF number corresponding to wheel rim displacements. Matrices $\mathbf{B}_{D,BF}$ and $\mathbf{K}_{D,BF}$ describe the support of engine stators with gear housings to the bogie frame in silent blocks. Matrices $\mathbf{B}_{W,BF}$ and $\mathbf{K}_{W,BF}$ describe dampings and stiffnesses of the primary suspension at points T_7 to T_{10} (damping) and P_5, P_6, P_9, P_{10} (stiffness) and the longitudinal wheelset guide between journal boxes and the bogie frame at points P_7, P_8, P_{11}, P_{12} (see Fig. 1). The vector $\mathbf{f}_G$ expresses all gravitational forces and the vector $\mathbf{f}_E(\dot{\boldsymbol{q}})$ expresses the engine driving torques. The vector $\mathbf{f}_{R,W}(\mathbf{q}, \dot{\boldsymbol{q}}, t)$ includes contact forces between rails and wheel rims depends on operational parameters, displacements and velocities of wheel rims and track or wheel surface deviations $\Delta_j(t), j = 1, 2, 3, 4$.

3. Linearized mathematical model of the bogie

To optimize bogie design parameters we neglect lateral track irregularities. The engine torques and creep characteristics will be linearized in the neighbourhood of the static equilibrium. Let us suppose an operational state of the railway vehicle running along the kinematically exact straight track in static equilibrium which is given by longitudinal creepage s_0 of all kinematically exact rim wheels, by forward vehicle velocity v and by vertical wheel forces N_0. To all mentioned operational parameters correspond engine torques and only longitudinal creep forces at the contact between rails and wheels given by

$$M(s_0, v) = 2\mu_0 N_0 r_0 \frac{1}{p}\,,\ T_0 = \mu_0 N_0\,, \tag{10}$$

where $\mu_0 = \mu(s_0, v)$ is longitudinal creep coefficient [2, 8], $p = \frac{\omega_E}{\omega_w}$ is speed ratio and r_0 is the wheel radius in the central position. If the static equilibrium is disturbed by any possible excitation sources, the bogie vibrates and the vector of generalized coordinates can be expressed as a sum of static and dynamic (perturbance) displacements

$$\boldsymbol{q}(t) = \mathbf{q}_0 + \Delta\boldsymbol{q}(t)\,, \tag{11}$$

where before the disturbance, the velocity vector $\dot{\boldsymbol{q}}_0$ has only nonzero coordinates corresponding to rotation of wheelset drive components. Linearized engine torque characteristics, longitudinal creep coefficients and vertical wheel forces can be expressed as

$$M = M_0(s_0, v) - b_E \Delta\dot{\varphi}_1\,, \tag{12}$$

$$\mu(s_i, v) = \mu_0 + \left[\frac{\partial\mu}{\partial s_i}\right]_{s_i = s_0} (s_i - s_0)\,,\ i = 12, 14\,, \tag{13}$$

$$N_i = N_0 + m_R(\ddot{\Delta}_j - \Delta\ddot{v}_i^w) + b_R(\dot{\Delta}_j - \Delta\dot{v}_i^w) + k_R(\Delta_j - \Delta v_i^w)\,, \tag{14}$$

where values denoted with Δ correspond to perturbance general coordinates $\Delta\boldsymbol{q}(t)$. The subscript $j = 1$ corresponds to wheelset node $i = 12$ and $j = 2$ to $i = 14$ of ID1 and $j = 3$ to $i = 12$ and $j = 4$ to $i = 14$ of ID2, respectively. Longitudinal creepages of wheels are defined by

$$s_i = s_0 + \frac{\pm\Delta\dot{w}_i^w \mp r_i \Delta\dot{\varphi}_i^w}{v}\,,\ i = 12, 14\,. \tag{15}$$

Upper signs correspond to wheelset of ID1 and lower signs to wheelset ID2. Lateral creep forces $A_{i\,ad}$ and spin torque $M_{i\,ad}$ are calculated using Kalker's coefficients [5] computed for constant vertical wheel force N_0 in [9].

After expressing the engine torques according to (12) and linearization of longitudinal creep forces $T_{i\,ad} = \mu(s_i, v_0)N_i$ using (13) to (15) for $r = r_0$, $\Delta\dot{\boldsymbol{q}}(t) = \mathbf{0}$, $\Delta_j = 0$ the vectors of all external forces on right side of equations (8) can be written in the following form

$$\mathbf{f}_G + \mathbf{f}_E(\dot{\boldsymbol{q}}) + \mathbf{f}_{R,W}(\mathbf{q}, \dot{\boldsymbol{q}}, t) = \mathbf{f}_0 - [\mathbf{B}_E + \mathbf{B}_{ad}(s_0, v)]\Delta\dot{\boldsymbol{q}}(t) + \mathbf{f}(t)\,. \tag{16}$$

The vector $\mathbf{f}_0 = \mathbf{K}\mathbf{q}_0$ expresses static force effects before the disturbance by track or wheel surfaces irregularities. The diagonal matrix $\mathbf{B}_E$ is determined by the inclination b_E of engine torques characteristics and the nonsymmetrical matrix $\mathbf{B}_{ad}(s_0, v)$ express the influence of the linearized creep forces between wheels and rails. The creep forces depend on longitudinal creepage s_0 defining the equilibrium state before the disturbance and on the vehicle velocity v. The excitation vector $\mathbf{f}(t)$ has non-zero components f_j on position corresponding to vertical displacements of the wheel rims v_i^w, $i = 12, 14$ for ID1 and ID2 in the general coordinate vector $\boldsymbol{q}(t)$ (here 81, 87, 179, 185). They fulfill

$$f_j = m_R \ddot{\Delta}_j(t) + b_R \dot{\Delta}_j(t) + k_R \Delta_j(t)\,,\; j = 1, 2, 3, 4\,. \tag{17}$$

The model (8) using (11) and (16) can be written in perturbance coordinates $\Delta\boldsymbol{q}(t)$ in the neighbourhood of the static equilibrium as

$$\mathbf{M}\Delta\ddot{\boldsymbol{q}}(t) + [\mathbf{B} + \mathbf{B}_E + \mathbf{B}_{ad}(s_0, v)]\Delta\dot{\boldsymbol{q}}(t) + \mathbf{K}\Delta\boldsymbol{q}(t) = \mathbf{f}(t)\,. \tag{18}$$

4. Optimization in term of dynamic response excited by track irregularities

The track irregularities can be understood as stationary stochastic process described by spatial power spectral density (PSD) function [6] $S(F)$ depends on spatial frequency $F = \frac{1}{\lambda}$ given in cycle parameter (λ is a wavelength). Several track measurements along the track have shown that $S(F)$ can be approximately expressed in the log-log coordinate system by piecewise straight line [1] in the analytical form

$$S(F) = S_i \left(\frac{F}{F_i}\right)^{\kappa_i}\,,\; F \in \langle F_i, F_{i+1}\rangle,\; \kappa_i = \frac{\log\frac{S_{i+1}}{S_i}}{\log\frac{F_{i+1}}{F_i}} \tag{19}$$

and S_i (S_{i+1}) are PSD values for spatial frequencies F_i (F_{i+1}).

The spatial PSD must be transformed into the standard PSD depending on frequency of the waves $f = vF$ [Hz] in the form [5]

$$S(f) = \frac{1}{v} S_i \left(\frac{f}{vF_i}\right)^{\kappa_i}\,,\; f \in \langle f_i, f_{i+1}\rangle\,, \tag{20}$$

where $v\,[\mathrm{ms}^{-1}]$ is the vehicle forward velocity.

The linearized model (18) can be rewritten after Fourier transformation into the frequency domain

$$\{-\omega^2\mathbf{M} + \mathrm{i}\omega[\mathbf{B} + \mathbf{B}_E + \mathbf{B}_{ad}(s_0, v)] + \mathbf{K}\}\Delta\mathbf{q}(\omega) = z_R(\omega)\boldsymbol{\Delta}(\omega)\,, \tag{21}$$

where in accordance with (17) the complex reduced track stiffness is

$$z_R = -\omega^2 m_R + \mathrm{i}\omega b_R + k_R \tag{22}$$

and for the time shift $\Delta t = \frac{l}{v}$, determined by vehicle velocity v and wheelbase of the bogie l, the vector of Fourier transformations of the rail deviations is

$$\boldsymbol{\Delta}(\omega) = [\ldots\Delta_1(\omega)\ldots\Delta_2(\omega)\ldots\Delta_2(\omega)\mathrm{e}^{\mathrm{i}\omega\Delta t}\ldots\Delta_1(\omega)\mathrm{e}^{\mathrm{i}\omega\Delta t}]^T\,. \tag{23}$$

The Fourier transformation of an arbitrary dynamic displacements is

$$\Delta q_i(\omega) = G_{i,1}(\omega)\Delta_1(\omega) + G_{i,2}(\omega)\Delta_2(\omega)\,,\; i = 1, \ldots, 189\,, \tag{24}$$

where the corresponding frequency response functions are

$$G_{i,1}(\omega) = z_R(\omega)[g_{i,81}(\omega) + g_{i,185}(\omega)\mathrm{e}^{\mathrm{i}\omega\Delta t}],\; G_{i,2}(\omega) = z_R(\omega)[g_{i,87}(\omega) + g_{i,179}(\omega)\mathrm{e}^{\mathrm{i}\omega\Delta t}] \tag{25}$$

and $g_{i,j}(\omega)$ are elements of frequency response function (FRF) matrix

$$\mathbf{G}(\omega) = \{-\omega^2\mathbf{M} + \mathrm{i}\omega[\mathbf{B} + \mathbf{B}_E + \mathbf{B}_{ad}(s_0, v)] + \mathbf{K}\}^{-1}\,. \tag{26}$$

The subscripts i correspond to general coordinates and subscript j (here 81, 87, 179, 185) to vertical displacements of the wheel rims in $\boldsymbol{q}(t)$. The vertical irregularities of the rails along the track can be understood as an ergodic Gaussian process with zero mean values [3] and the cross correlation between the rail irregularities Δ_1 and Δ_2 equates to zero.

The dynamic force vectors transmitted by silent blocks can be expressed in the form

$$\Delta\mathbf{f}_j(t) = \mathbf{K}_j\Delta\mathbf{q}_1(t) + \mathbf{B}_j\Delta\dot{\boldsymbol{q}}_1(t) - \mathbf{K}_{j\,BF}\Delta\mathbf{q}_{BF}(t) - \mathbf{B}_{j\,BF}\Delta\dot{\boldsymbol{q}}_{BF}(t)\,, \tag{27}$$

where matrices

$$\mathbf{K}_j = [\mathbf{K}_t\ \mathbf{K}_t\mathbf{R}_j^T]\,,\ \mathbf{K}_{j\,BF} = [\mathbf{K}_t\ \mathbf{K}_t\mathbf{R}_{j\,BF}^T]$$

are determined by diagonal stiffness matrix $\mathbf{K}_t$ of one silent block and skew-symmetric matrices $\mathbf{R}_j$ ($\mathbf{R}_{jBF}$) are determined by coordinates of elasticity centre of silent blocks in coordinate system x_1, y_1, z_1 (x_{BF}, y_{BF}, z_{BF}) and $\Delta\mathbf{q}_1(t)$ ($\Delta\mathbf{q}_{BF}(t)$) is the vector of disturbance coordinates of mass centre S_1 (S_{BF}). Providing that the damping of silent blocks is proportional with coefficient β, the Fourier transform of the $\Delta\mathbf{f}_j(t)$ is

$$\Delta\mathbf{f}_j(\omega) = (1 + \mathrm{i}\omega\beta)[\mathbf{K}_j\Delta\mathbf{q}_1(\omega) - \mathbf{K}_{j\,BF}\Delta\mathbf{q}_{BF}(\omega)] \tag{28}$$

and can be rewritten, with regard to (24), into

$$\Delta\mathbf{f}_j(\omega) = \mathbf{g}_{j,1}(\omega)\Delta_1(\omega) + \mathbf{g}_{j,2}(\omega)\Delta_2(\omega)\,,$$

where $\mathbf{g}_{j,1}(\omega)$ and $\mathbf{g}_{j,2}(\omega)$ are vectors of frequency response functions of dimension 3. Their PSD are

$$\mathbf{S}_j = \mathbf{G}_j(\omega)\mathbf{S}_\Delta(\omega)\mathbf{G}_j^H(\omega)\,,\ j = A_1, B_1, C_1, A_2, B_2, C_2\,, \tag{29}$$

where

$$\mathbf{G}_j(\omega) = [\mathbf{g}_{j,1}(\omega)\ \mathbf{g}_{j,2}(\omega)] \in C^{3,2}\,,\ \mathbf{S}_\Delta(\omega) = \mathrm{diag}\,(S_{\Delta_1}(\omega), S_{\Delta_2}(\omega))$$

and superscript H denotes the transposition of conjugate matrix.

Let the aim of a optimization of the design parameters $\mathbf{p} = [p_i]$ be a suppression of the forces transmitted by the silent blocks. The sufficient objective function is

$$\psi(\mathbf{p}) = \sqrt{\sum_j \sum_k \boldsymbol{S}_j^T(\mathbf{p}, f_k)\boldsymbol{S}_j(\mathbf{p}, f_k)}\,, \tag{30}$$

where the frequency range in Hz is divided with chosen step Δf [Hz] and holds

$$f_k = f_{min} + k\Delta f\,,\ k = 0, 1, \ldots, \frac{f_{max} - f_{min}}{\Delta f}\,. \tag{31}$$

Especially a minimization of vertical wheel forces has a dominant interest to vertical dynamic of railway vehicles. According to (14) the Fourier transform of their dynamic components acting to wheelset ID1 is

$$\Delta N_i(\omega) = z_R(\omega)[\Delta_j(\omega) - \Delta v_i^w(\omega)] \text{ for } i = 12, j = 1 \text{ and } i = 14, j = 2\,. \tag{32}$$

According to (25) frequency response functions for instance between vertical wheel forces acting in node 12 on the wheelset of ID1 and rail vertical irregularities can be written as

$$\bar{G}_{12,1}(\omega) = z_R(\omega)[1 - G_{81,1}(\omega)]\,,\ \bar{G}_{12,2}(\omega) = -z_R(\omega)G_{81,2}(\omega)\,, \tag{33}$$

where subscript 81 corresponds to vertical displacement v_{12}^{w} of the wheel rim with the centre in mode 12. The PSD of the vertical wheel forces depending on frequency $f = \frac{\omega}{2\pi}$ in Hz are calculated on the basis of PSD vertical rail irregularities in the form

$$S_{N_i}(f) = S_{\Delta_1}(f)|\bar{G}_{i,1}|^2 + S_{\Delta_2}(f)|\bar{G}_{i,2}|^2\,,\ i = 12, 14\,. \tag{34}$$

The upper limits of state dynamic values can be calculated by algebraic sum of static values and corresponding standard deviations as follows [11]

$$N_i = N_0 + \xi\sigma_{N_i},\ \text{where}\ \sigma_{N_i}^2 = 2\int_0^{\infty} S_{N_i}(f)\mathrm{d}f,\ \xi = 2 \div 3\,, \tag{35}$$

the sufficient objective function for minimization of vertical wheel forces is

$$\psi(\mathbf{p}) = \sum_i \sum_k S_{N_i}(\mathbf{p}, f_k) \tag{36}$$

in the frequency range defined in (31).

5. Optimization in term of dynamic response excited by polygonalized running surface of the wheels

The polygonalization process forms radial irregularities of the running surface of the wheels which can be described by harmonic components with frequencies [3]

$$\omega_m = \frac{vm}{r}\,,\ m = 2, 3, \dots\,, \tag{37}$$

where v is the vehicle forward velocity, r is the averaged wheel radius and m is so called order of the wheel polygonalization. The excitation vector in model (18) caused by polygonalization can be expressed in the complex form

$$\mathbf{f}(t) = \sum_m z_R(\omega_m)\boldsymbol{\Delta}_m \mathrm{e}^{\mathrm{i}\omega_m t}\,, \tag{38}$$

where the complex reduced track stiffness is given by expression (22) for $\omega = \omega_m$ and the vector of complex amplitudes of irregularities is

$$\boldsymbol{\Delta}_m = [\dots \Delta_{1m}\mathrm{e}^{\mathrm{i}\delta_{1m}} \dots \Delta_{2m}\mathrm{e}^{\mathrm{i}\delta_{2m}} \dots \Delta_{3m}\mathrm{e}^{\mathrm{i}\delta_{3m}} \dots \Delta_{4m}\mathrm{e}^{\mathrm{i}\delta_{4m}} \dots]^T\,. \tag{39}$$

Radial irregularities of the wheels corresponding to polygonalization of order m are determined by their amplitudes Δ_{jm} and phases δ_{jm}. Non-zero components of vectors $\boldsymbol{\Delta}_m$ are on positions corresponding to vertical displacements of the wheel rims in $\boldsymbol{q}(t)$.

The steady dynamic bogie displacements in the complex form are

$$\Delta\boldsymbol{q}(t) = \sum_m \mathbf{q}(\omega_m)\mathrm{e}^{\mathrm{i}\omega_m t} \tag{40}$$

with complex amplitudes

$$\mathbf{q}(\omega_m) = z_R(\omega_m)\mathbf{G}(\omega_m)\mathbf{\Delta}_m = [q_i(\omega_m)]\,, \tag{41}$$

where FRF matrix has the form (26) for $\omega = \omega_m$. The steady dynamic force vectors transmitted by silent blocks according to (27) and (28) are expressed as

$$\Delta\mathbf{f}_j(t) = \sum_m (1 + \mathrm{i}\omega_m\beta)[\mathbf{K}_j\mathbf{q}_1(\omega_m) - \mathbf{K}_{j\,BF}\mathbf{q}_{BF}(\omega_m)]\mathrm{e}^{\mathrm{i}\omega_m t}\,, \tag{42}$$

where $\mathbf{q}_1(\omega_m)$ and $\mathbf{q}_{BF}(\omega_m)$ are subvectors of $\mathbf{q}(\omega_m)$ corresponding to mass centres of the engines and mass centre of the bogie frame displacements.

The steady dynamic vertical wheel forces according to (14) are expressed as

$$\Delta N_i(t) = \sum_m z_R(\omega_m)[\Delta_{jm}\mathrm{e}^{\mathrm{i}\delta_{1m}} - \Delta v_i^w(\omega_m)]\mathrm{e}^{\mathrm{i}\omega_m t},\ i = 12,\ j = 1 \text{ and } i = 14\, j = 2\,. \tag{43}$$

From the viewpoint of the dynamic loading of the silent block j ($j = A_1, B_1, C_1, A_2, B_2, C_2$) the sufficient objective function can be formulated as the weighted sum of the global forces transmitted by the chosen silent block

$$\psi(\mathbf{p}) = \sum_m g_m \sum_j \sum_k \sqrt{\mathbf{f}_j^H(\mathbf{p}, m, v_k)\mathbf{f}_j(\mathbf{p}, m, v_k)}\,, \tag{44}$$

where in accordance with (42) the force vectors of complex amplitudes

$$\mathbf{f}_j(\mathbf{p}, m, v_k) = (1 + \mathrm{i}\omega_m^{(k)}\beta)[\mathbf{K}_j\mathbf{q}_1(\mathbf{p}, \omega_m^{(k)}) - \mathbf{K}_{j\,BF}\mathbf{q}_{BF}(\mathbf{p}, \omega_m^{(k)})] \tag{45}$$

are calculated in the frequency range

$$\omega_m^{(k)} = \frac{v_k m}{r}\,,\ v_k = 0, 1, \ldots, \frac{v_{max} - v_{min}}{\Delta v} \tag{46}$$

and for current optimization parameters $\mathbf{p}$.

Analogous to minimization of the silent block dynamic loading we can formulate the objective function for the minimization of the vertical wheel forces

$$\psi(\mathbf{p}) = \sum_m g_m \sum_i \sum_k |N_i(\mathbf{p}, m, v_k)|\,, \tag{47}$$

where in accordance with (43) complex amplitudes of the forces

$$N_i(\mathbf{p}, m, v_k) = z_R(\omega_m^{(k)})[\Delta_{jm}\mathrm{e}^{\mathrm{i}\delta_{jm}} - v_i^w(\mathbf{p}, \omega_m^{(k)})]\,,\ i = 12,\ j = 1 \text{ and } i = 14,\ j = 2 \tag{48}$$

are calculated in the frequency range defined in (46) and for current optimization parameters $\mathbf{p}$.

6. Application

The presented methodology and developed software in MATLAB code was tested, among other problems, for minimization of the vertical wheel forces acting on the wheelset of ID1. Four design parameters

$$\mathbf{p} = [k_y^w, k_{yy}^w, b_R, k_R]^T \tag{49}$$

were chosen as the optimization parameters. They were constrained by lower and upper ratios limits (marked by bar) with respect to reference values $k_y^w = 7 \cdot 10^9\,\mathrm{Nm}^{-1}$ (radial stiffness of all wheels), $k_{yy}^w = 10^8\,\mathrm{Nm\,rad}^{-1}$ (bending stiffness of all wheels), $b_R = 8 \cdot 10^4\,\mathrm{Nm}^{-1}\,\mathrm{s}$ (damping coefficient of the track structure [6]), $k_R = 8 \cdot 10^7\,\mathrm{Nm}^{-1}$ (stiffness of track structure [6]). Applied constraints are

$$\bar{\mathbf{p}}_L < \bar{\mathbf{p}} < \bar{\mathbf{p}}_U\,, \tag{50}$$

where $\bar{\mathbf{p}}_L = [0.5, 0.5, 0.2, 0.2]^T$ and $\bar{\mathbf{p}}_U = [2, 2, 5, 5]^T$.

The first presented optimization problem was defined by the objective function in the form (36) for the excitation by track vertical irregularities described by spatial PSD of left $S_{\Delta_1}(F)$ and right $S_{\Delta_2}(F)$ rails measured along the track lengthwise 4 km with step 0.5 m obtained in cooperation with ŠKODA TRANSPORTATION s.r.o. [12]. The coordinates of the breakpoints of the piecewise straight lines approximating the mentioned PSD are introduced in [11] (Tab. 1). We assume the operational parameters $s_0 = 0.002$; $v = 200\,\mathrm{km/h}$ and $N_0 = 10^5$ N corresponding to bogie's static equilibrium before disturbance by running of the bogie frame at the real track. The frequency range of the PSD in the objective function (36) was chosen according to (31) $f_{min} = 1\,\mathrm{Hz}$, $f_{max} = 50\,\mathrm{Hz}$ with step $\Delta f = 1\,\mathrm{Hz}$.

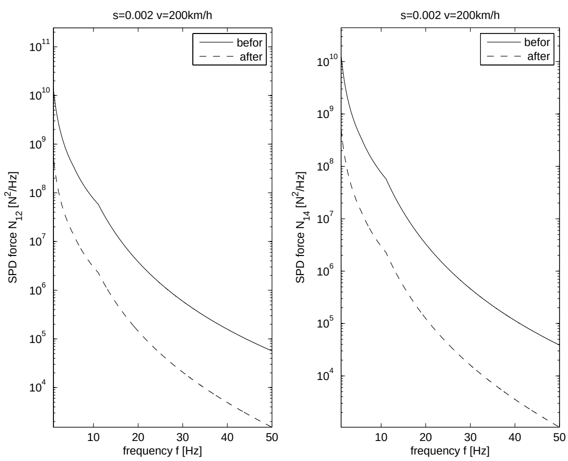

Fig. 3. Power spectral densities of the vertical wheel forces

As an illustration in Fig.3 we show the PSD of the vertical wheel forces N_{12} and N_{14} before and after optimization. The corresponding values of the objective functions, standard deviations and relative optimization parameters before and after optimization are summarized in Table 1.

Table 1. Starting and achieved values of the vertical wheel forces optimization for excitation by track irregularity

state	$\psi(\mathbf{p})$	σ_{N_1}	σ_{N_2}	$\overline{k_y^w}$	$\overline{k_{yy}^w}$	$\overline{b_R}$	$\overline{k_R}$
before	$8.98 \cdot 10^9$	$2.35 \cdot 10^5$	$1.68 \cdot 10^5$	1	1	1	1
after	$3.58 \cdot 10^8$	$0.47 \cdot 10^5$	$0.34 \cdot 10^5$	1	2	0.2	0.2

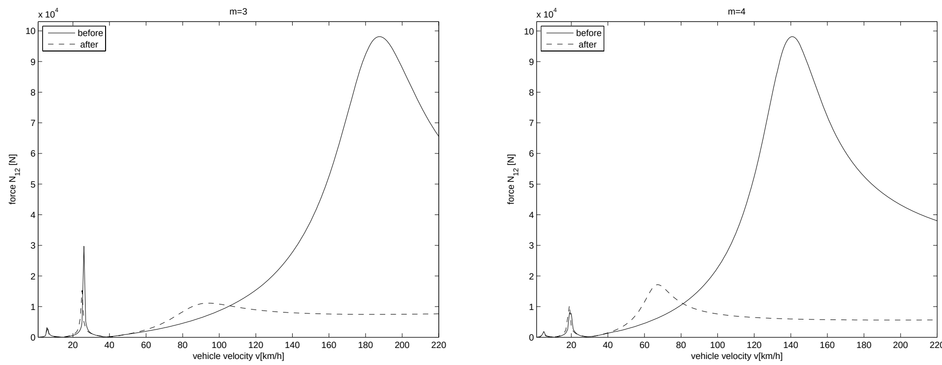

Fig. 4. Amplitude characteristics of the vertical wheel forces

The second presented optimization problem was defined by the objective function in the form (47) for the excitation by polygonalized wheel running surfaces and vertical wheel forces N_{12} and N_{14} acting on the wheelset of ID1. Measurements made by the German railroad company Deutsche Bahn [3] show that dominant polygonalizations are of 3^{rd} and 4^{th} order and can be approximated by harmonic functions with an amplitude Δ_{jm} =0.3 [mm] and for $\delta_{jm} = 0$ $(j = 1, 2, 3, 4)$. A method and software testing has been developed for weight coefficients $g_3 = 1$ and $g_4 = 1$ separately (other weight coefficients are zero) in velocity range $v_{min} = 50$, $v_{max} = 200$, $\Delta v = 2$ [km/h].

As an illustration, in Fig. 44 we show the amplitude characteristics of the vertical wheel force N_{12} before and after optimization for polygonalization of 3^{rd} order and 4^{th} order. The corresponding values of the objective functions, maximal values of vertical wheel force and relative optimization parameters before and after optimization are summarized in Tab. 2. Main resonances characterized by vertical wheelset vibrations on vehicle velocity $v \approx 183$ [km/h] (for $m = 3$) and $v \approx 137$ [km/h] (for $m = 4$) were eliminated.

The minimization of the objective function (44) and (47) was realized in code MATLAB by a simplex method. The total computational time (at Workstation HP xw4300) for four optimization parameters defined in (49) was approximately 80 s. The computational time increases approximately with square power of the optimization parameter number.

Table 2. Starting and achieved values of the vertical wheel forces optimization for excitation by wheel polygonalization

value	m=3		m=4	
	before	after	before	after
$\psi(\mathbf{p})$	$4.96 \cdot 10^6$	$1.15 \cdot 10^6$	$6.56 \cdot 10^6$	$1.15 \cdot 10^6$
$N_{12\,max}$	$9.6 \cdot 10^4$	$1.1 \cdot 10^4$	$9.6 \cdot 10^4$	$1.6 \cdot 10^4$
$N_{14\,max}$	$9.4 \cdot 10^4$	$1.2 \cdot 10^4$	$9.4 \cdot 10^4$	$1.7 \cdot 10^4$
$\overline{k_y^w}$	1	1.99	1	0.5
$\overline{k_{yy}^w}$	1	0.5	1	0.59
$\overline{b_R}$	1	.80	1	0.47
$\overline{k_R}$	1	0.2	1	0.2

7. Conclusion

This paper presents an original methodology of parametric optimization of the railway vehicle bogie with elastic wheels. The approach to optimization is based on the mathematical modelling of the bogie in perturbance coordinates with respect to operational state of static equilibrium and on the calculation of the dynamic response caused by geometric irregularities of the track or running wheel surfaces. Two types of objective functions has been formulated for the problem of dynamic load minimization. The first type is based on power spectral density functions and the second type on amplitude characteristics of the dynamic forces transmitted by rubber silent blocks between engine stators with gear housings and the bogie frame and the dynamic forces in contact between rails and wheels in a vertical direction. These couplings transfer great dynamic forces caused by considered kinematic excitation which essentially influences a service live of the wheelsets, rails and support of engine stators.

The developed software in MATLAB code enables to choose different design parameters as the optimization parameters and a minimization of arbitrary displacements and accelerations of bogie components and forces transmitted by different couplings. In a close future, the multicriterial objective functions will be used for optimization of bogie design parameters.

Acknowledgements

This paper includes a methodology of an optimization developed within the framework of the research project MSM 4977751303 and application results from the Research Project 1M0519 — Research Centre of Rail Vehicles supported by the Czech Ministry of Education.

References

[1] M. Balda, Study of behaviour of the railway bogie with nonlinear couplings during driving along random track irregularities, Inaugural disertation, University of West Bohemia, Plzeň, 1993, (in Czech).

[2] M. Byrtus, V. Zeman, Z. Hlaváč, Modelling and modal properties of an individual wheelset drive of a railway vehicle, Sborník XVIII. mezinárodné konferencie Súčasné problémy v kolajových vozidlách – PRORAIL2007, Žilina.

[3] H. Claus, W. Schiehlen, System Dynamics of Railcars with Radial and Laterelastic Wheels, in System Dynamics and Long Term Behaviour of Railway Vehicles, Track and Subgrade (K. Popp and W. Schiehlen eds.), Springer, 2003, pp. 65–84.

[4] V. Feldman, E. Kreuzer, F. Pinto, V. Schlegel, Monitoring the dynamic of railway track by means of the Karhunen-Loeve-transfomation, in System Dynamics and Long Term Behaviour of Railway Vehicles, Track and Subgrade (K. Popp and W. Schiehlen eds.), Springer, 2003, pp. 231–246.

[5] V. Garg, R. Dukkipati, Dynamics of railway vehicle systems, London Academic Press, 1984.

[6] K. Knothe, M. Yu, H. Ilias, Measurement and Modelling of Resilient Rubber Rail-Pads, in System Dynamics and Long Term Behaviour of Railway Vehicles, Track and Subgrade (K. Popp and W. Schiehlen eds.), Springer, 2003, pp. 265–274.

[7] K. Popp, W. Schiehlen, Fahrzeugdynamik, B. G. Teubner, 1993.

[8] V. Zeman, Z. Hlaváč, M. Byrtus, Modelling and modal properties of a railway vehicle wheelset drive with a hollow shaft, Proceedings of the Conference Engineering Mechanics 2007, Svratka, pp. 38–39, (full paper on cd-rom).

[9] V. Zeman, Z. Hlaváč, M. Byrtus, Modellig and modal properties of the railway vehicle bogie with two individual wheelset drives, Applied and Computational Mechanics, 1 (1) (2007) 371–380.

[10] V. Zeman, Z. Hlaváč, M. Byrtus, Modelling of the wheelset drive vibration of the 109E type locomotive, Research report n. H2, H9-U6-01/2006, Research Centre of Rail Vehicles, Plzeň, 2007, (in Czech).

[11] V. Zeman, Z. Hlaváč, M. Byrtus, Dynamic analysis of the railway vehicle bogie, Proceedings of the Conference Engineering Mechanics 2008, Svratka, pp. 284–285, (full paper on cd-rom pp. 1 231–1 240).

[12] V. Zeman, Z. Hlaváč, M. Byrtus, J. Dupal, Modelling of the bogie vibration of the 109E type locomotive, Research report n. H2 + H9/2007, Research Centre of Rail Vehicles, Plzeň, 2008, (in Czech).