

# Examination of the Fatigue Life under Combined Loading of Specimens

F. Fojtík<sup>a,\*</sup>, J. Fuxa<sup>a</sup>

<sup>a</sup> Faculty of Mechanical Engineering, VŠB – Technical University of Ostrava, 17. listopadu 15, 708 33 Ostrava-Poruba, Czech Republic

Received 25 August 2008; received in revised form 6 October 2008

---

## Abstract

This contribution describes experimental results under combined loading of specimens manufactured from common construction steel 11523. Specimens were gradually loaded by amplitude of the torque, then by combination of torque and tension prestress. The last set of specimens was loaded in combination of torque and inner overpressure. To obtain the required input values the stress-strain analysis of specimens by finite element method in software Ansys was performed within the last experiment. For evaluation of the results the Fuxa's criterion was applied. The performed experiments and their results embody a good agreement with bellow mentioned conjugated strength criterion. The experiments were performed on reconstructed testing machine equipped by pressure chamber.

© 2008 University of West Bohemia in Pilsen. All rights reserved.

*Keywords:* combined loading, high-cycle fatigue, experiment, multi-axial fatigue, mean stress effect

---

## 1. Introduction

To verify the developed multi-axial Fuxa's conjugated strength criterion and to determine the proper constants the new jig was developed which generalizes the possibilities of reconstructed testing machine SHENCK typer PWXN [1]. In this case the testing machine was newly equipped by pressure chamber which makes possible to load the specimen by constant inner overpressure (in the range 0 to 70 MPa) in addition with torque amplitude. It is also possible to add the constant tension/pressure pre-stress into the system and to realize the proper combinations of stress states.

## 2. Experimental Device

For material testing under combined loading in the region of high-cycle fatigue and for verification of Fuxa's conjugated strength criterion was reconstructed the testing device SHENCK type PWXN [1, 2]. The new conception of the testing device changes the loading character of the specimen from deformation-controlled to force-controlled — see fig. 1.

The base of the testing machine is the frame 1 with gear box which is driven by asynchronous electromotor. Required torque is gained by the acceleration of driven part and by balance wheel located in measuring box 2. Axial tension/pressure force is gained by straight-line hydro motor 3 connected with hydraulic aggregate. Newly is the testing machine equipped by pressure chamber 4 which is connected with multiplier and hydraulic aggregate. The measured

---

\*Corresponding author. Tel.: +420 597 323 292, e-mail: frantisek.fojtik@vsb.cz.

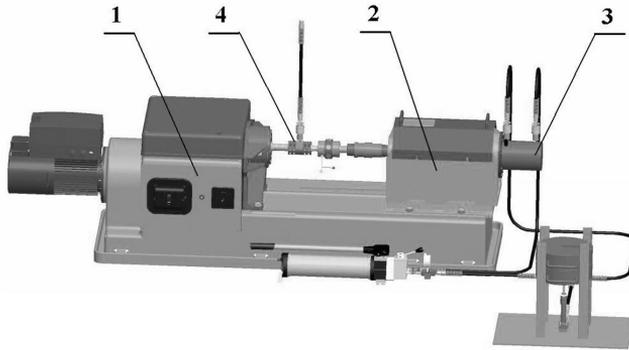


Fig. 1. Experimental device

values of the torque, axial force and inner overpressure are evaluated in software LabVIEW 8.0.1. [4].

The new conception of the testing device is patented as: Experimental Device for combined loading of specimens, Nr. 17286 (2007).

### 3. Experiment — alternating torsion

Specimens manufactured from steel 11523, melting nr. T18556 whose parameters are in fig. 2 were loaded by amplitude of torque in the condition of alternating cycle. The amplitude of torque was gradually decreased until the limit  $10^7$  cycles was reached.

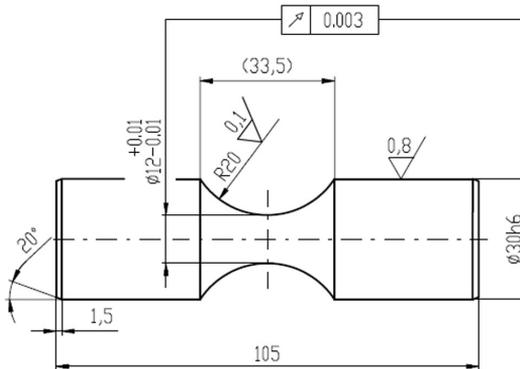


Fig. 2. Specimen

The results are placed in tab. 1. In the fig. 3 can be seen measured values, Baskin's approximation curves (1) and Fuxa's approximation curves (2) [5]. Point of crack initiation under static torsion was measured by reconstructed Testing machine INOVA in the institute lab.

Basquin's approximation:

$$\tau_a = \tau_f' \cdot (2N_f)^b, \quad (1)$$

where  $\tau_f'$  (758,75 MPa) is the fatigue strength and  $b$  (-0.0836) is fatigue strength coefficient.

Table 1. Experimental results for alternating torsion

Specimen Nr.	Stress amplitude $\tau_a$ [MPa]	Number of cycles	Notes
1	250	267 800	
2	224	1 238 000	
3	221	1 539 700	
4	215	1 620 000	
5	200.5	3 547 000	
6	196	10 103 000	No crack generated
7	193	10 356 000	No crack generated
8	546	0.25	

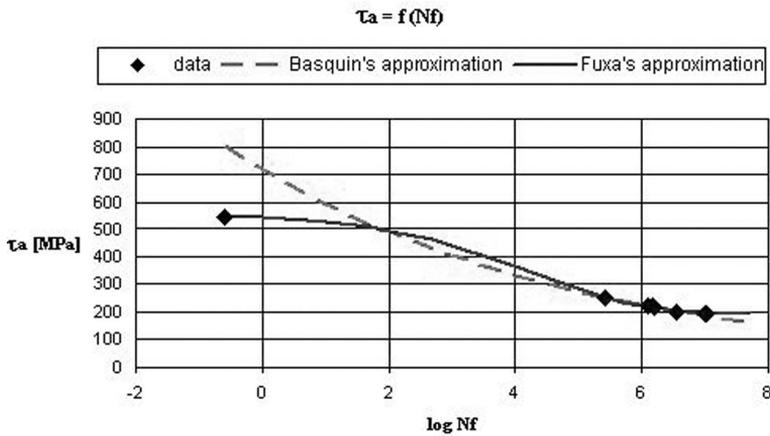


Fig. 3. W – curve for alternating torsion

Fuxa’s approximation:

$$\tau_{aF} = \frac{\tau_f + \tau_C}{2} + \frac{\tau_f - \tau_C}{2} \cdot \cos \left\{ \pi \cdot \left[ \frac{\log(4 \cdot N_f)}{\log(4 \cdot N_C)} \right]^{a_1} \right\}, \quad (2)$$

for  $N_f$  in interval  $[1/4; N_C]$  and  $\tau_{aF}$  in interval  $[\tau_f; \tau_C]$ .

$\tau_f$  (546 MPa) is a value of real shear strength,  $\tau_C$  (197 MPa) is the stress at the fatigue limit,  $N_C$  (9 130 000) is number of cycles at the fatigue limit,  $a_1$  (1.324) is constant,  $\tau_{aF}$  is the limit stress amplitude under alternating torsion and  $N_f$  is the limit number of cycles until crack initiation.

#### 4. Experiment — alternating torsion and tension prestress

The specimens according fig. 4 were used for this way of loading. Those specimens are manufactured from the same material and melting as in the previous case. Every series of specimens was loaded by different constant axial tension stress. For given tension stress value was chosen

the torque amplitude which was the specimen loaded until the crack initiation by. This amplitude was gradually decreased until the value when was the specimen able to endure  $10^7$  of cycles.

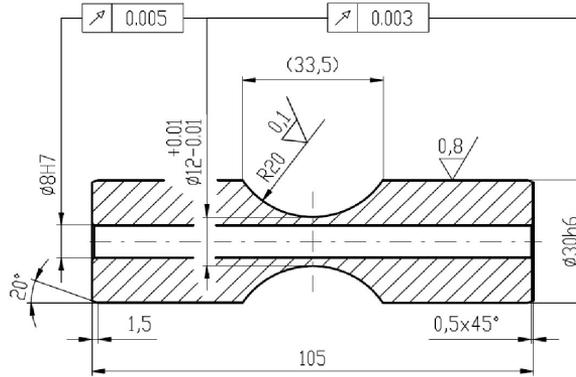


Fig. 4. Specimen for combined loading torsion — tension

The results of those experiments are mentioned in tab. 2. The experimental results are also shown in fig. 5 and approximated by lower described Fuxa's approximation (3, 4, 5, 6) which takes the influence of mean stress into account. Particular approximations are based on measured number of cycles which is mentioned in fig. 2.

Table 2. Experimental results for alternating torsion and tension prestress

Specimen Nr.	Tension Stress [MPa]	Stress amplitude $\tau_a$ [MPa]	Number of Cycles	Notes
1	110	174	649 000	
2	110	154.2	11 500 000	No crack generated
3	160.4	148	2 320 000	
4	160.4	139.5	11 600 000	No crack generated
5	228.4	203.7	28 640	
6	228.2	170	139 210	
7	228.4	120	10 800 000	No crack generated
8	310	116	1 141 000	
9	310	104.7	11 300 000	No crack generated

On the fig. 6 are displayed three phases of crack propagation for described combined loading.

Fuxa's approximation with influence of mean stress:

$$\tau_{aF2} = \frac{\tau_f^* + \tau_C^*}{2} + \frac{\tau_f^* - \tau_C^*}{2} \cdot \cos \left\{ \pi \cdot \left[ \frac{\log(4 \cdot N_f)}{\log(4 \cdot N_C)} \right]^{a_1} \right\}, \quad (3)$$

$$\tau_f^* = \frac{1}{\sqrt{3}} \cdot \left( (\sqrt{3} \cdot \tau_f)^2 - 2 \cdot \sqrt{3} \cdot \tau_f \cdot B_O \cdot \frac{\sigma_t}{3} + \sigma_t^2 \cdot \frac{B_O^2}{9} - \sigma_t^2 \right)^{1/2}, \quad (4)$$

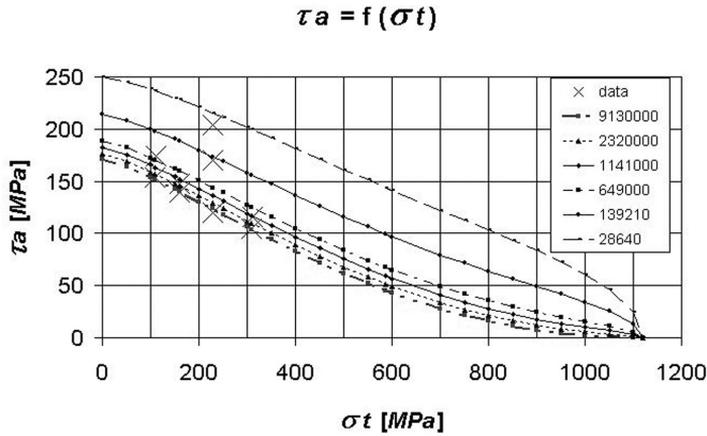


Fig. 5. Fuxa's approximation for combined torsion — tension loading

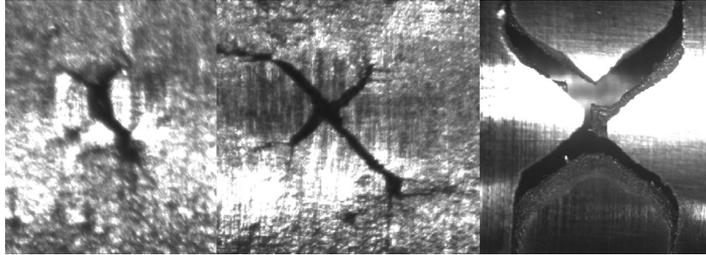


Fig. 6. Three phases of crack propagation for combined torsion — tension loading

where (4) is the static strength condition for  $N_f = 1/4$  and constant  $B_O$  is equal to:

$$B_O = 3 \cdot \left( \sqrt{3} \cdot \frac{\tau_f}{\sigma_f} - 1 \right), \quad (5)$$

$$\tau_C^* = \frac{\tau_C}{2} \cdot \left\{ 1 + \cos \left[ \pi \cdot \left( \frac{\sigma_t}{\sigma_f} \right)^B \right] \right\} \text{ is the strength condition for } N_f = N_C. \quad (6)$$

In the relations (3, 4, 5, 6) the  $\sigma_f$  is the real tension strength value,  $\tau_f$  is a value of real shear strength,  $\tau_C$  is the stress at the fatigue limit,  $N_C$  is number of cycles at the fatigue limit,  $a_1$  and  $B$  are constants,  $\tau_{aF2}$  is the limit amplitude of shear stress,  $\sigma_t$  is the constant tension stress and  $N_f$  marks the limit number of cycles until crack initiation.

## 5. Experiment — alternating torsion and inner overpressure

For this way of loading was the above mentioned testing machine equipped by a new type of specimen fixation which makes possible to use the overpressure chamber. This chamber is connected with multiplier and with hydraulic aggregate which serves for gaining of constant inner overpressure 0–70 MPa. The specimens described in fig. 7 were used for this type of loading. Those specimens were manufactured from the same material and melting as in previous cases. Every series of specimens was loaded by different constant overpressure. For given

constant inner overpressure value was chosen the torque amplitude which was the specimen loaded until the crack initiation by. This amplitude was gradually decreased until the value when was the specimen able to endure  $10^7$  of cycles. The results of those experiments are mentioned in tab. 3.

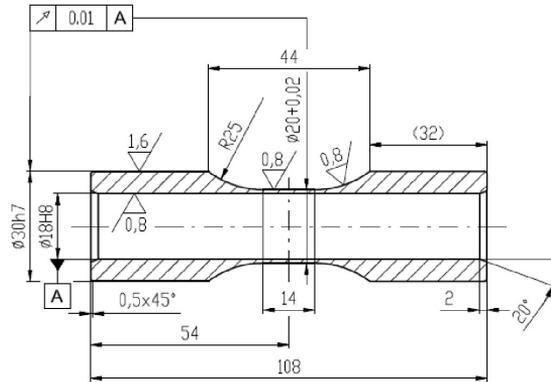


Fig. 7. Specimen for combined torsion — inner overpressure loading

Table 3. Experimental results for alternating torsion and tension prestress

Specimen Nr.	Overpressure [MPa]	$\sigma_t = \sigma_a + \sigma_{t1}$ [MPa]	Stress amplitude [MPa]	Number of cycles	Notes
1	13	155.1	158.3	496 900	
2	13	155.1	142.5	11 640 000	No crack generated
3	27	321.1	149	204 000	
4	27	321.1	142.5	311 560	
5	27	321.1	104.6	2 013 000	
6	27	321.1	91.6	10 118 800	No crack generated

From the evaluation of results can be observed the significant value of circumferential stress  $\sigma_{t1}$  and of axial stress  $\sigma_a$  at the surface of specimen. It is not easy to determine those two stresses analytically due to the complicated shape of specimen and face of acting inner overpressure. Hence this stress state had to be determined by finite element method in software ANSYS.

The static analysis was performed, where 1/4 of specimen was modeled, the SOLID element 186 was used, the material parameters were obtained on the basis of tensile test. The boundary conditions are chosen so that the resting 3/4 of specimen is compensated by symmetry and further one point of specimen face is fixed in three directions ( $x, y, z$ ). Opposite end of the specimen is free. On the relevant length the inner overpressure was applied. This length results from the dimensions of testing jig. Results for given overpressure are in tab. 3. Results of circumferential and axial stresses in MPa, obtained from FEM analysis for inner overpressure 13 MPa, are on fig. 8.

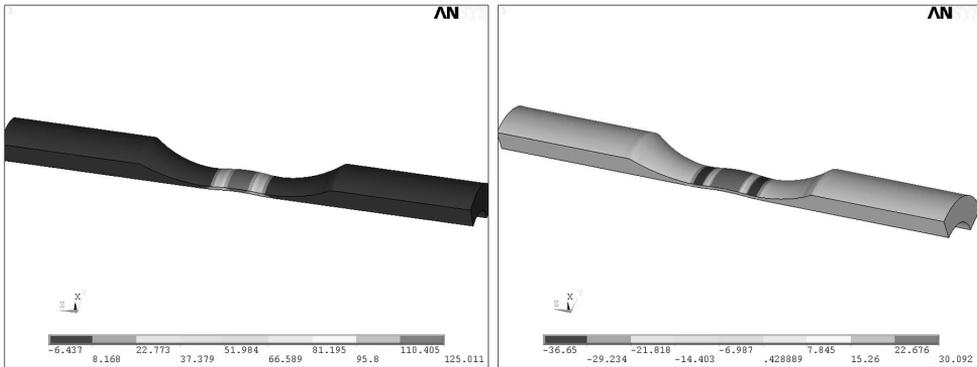


Fig. 8. Circumferential and axial stresses for inner overpressure 13 MPa

Experimental results obtained from performed experiments and computation are on fig. 9. Those results are approximated by already mentioned Fuxa’s approximation (3, 4, 5, 6) which takes into account the influence of mean stress. For from inner overpressure obtained stress state it is necessary to adjust the relations (4) Mentioned equation can be written as follows:

$$\tau_f^* = \frac{1}{\sqrt{3}} \cdot \left( \frac{(\sqrt{3} \cdot \tau_f)^2 - 2 \cdot \sqrt{3} \cdot \tau_f \cdot B_O \cdot (\sigma_{t1} + \sigma_a) / 3}{+(\sigma_{t1} + \sigma_a)^2 \cdot B_O^2 / 9 - \sigma_{t1}^2 + \sigma_{t1} \cdot \sigma_a + \sigma_a^2} \right)^{1/2} \quad (7)$$

This equation is based on reference stresses [3] and on the stress state analysis for given loading case.

Particular approximations result from measured number of cycles written in tab. 3. On the base of fig. 9 can be compared the measured data at the fatigue limit obtained from previous experiment. The good agreement with proposed approximation can be seen. On the fig. 10 are mentioned three different types of failures and the crack propagation for different amplitudes of loading and reached number of cycles.

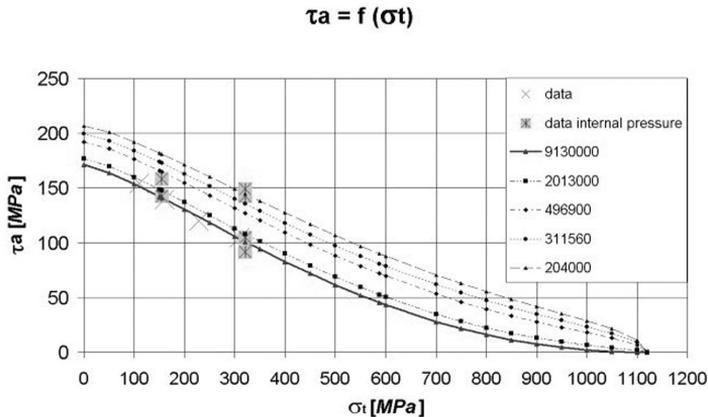


Fig. 9. Fuxa’s approximation for combined torsion — inner overpressure loading

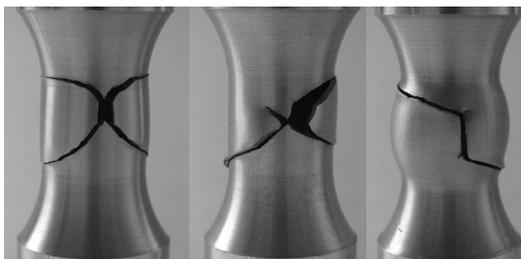


Fig. 10. Three types of crack propagations for different ways of loadings

## 6. Conclusion

The reconstructed testing machine equipped by overpressure chamber is briefly described in this contribution. Further are mentioned three types of experiments performed at the specimens made from steel 11523 melting T18556.

First experiment — alternating torsion. Obtained results are approximated by Basquin's and Fuxa's approximation. The Fuxa's approximation embodies a good agreement with experiment. The Basquin's approximation does not take into account the value of real shear strength — see Fig. 3.

Second experiment — combined loading by amplitude of torque and by constant axial tension force. The results are successfully approximated by Fuxa's approximation with the influence of mean stress. Also here the Fuxa's approximation embodies the good agreement with experiment — see Fig. 5. The constants of strength criterion were tuned on this experiment.

Third experiment — combined loading by the amplitude of torque and by inner overpressure. Appropriate circumferential and axial stresses (mentioned in Tab. 3) are obtained by finite element method and the modification of Fuxa's approximation is mentioned. The experimental results are approximated by Fuxa's approximation whose constants result from previous experiment. The good agreement can be seen here and hence it is possible to state the appropriate constant tuning for further possible combined loading — see fig. 9. Described Fuxa's approximation is a part of conjugated stress criterion which is based on the conception of reference normal and reference shear stresses, see [3] for more details.

## Acknowledgements

The paper was created under support of GACR, project no: 101/08/P141.

## References

- [1] F. Fojtík, The Experimental Machine for the Multiaxis Fatigue Testing of Material, Dissertation Thesis, FS VŠB-TU Ostrava, 2007.
- [2] F. Fojtík, The Fatigue Testing Machine and Experiment, In: *Applied Mechanics 2008*, Wisla, Polsko, 2008.
- [3] J. Fuxa, R. Kubala, F. Fojtík, Idea of Conjugated Strength, In: *Experimental Stress Analysis 2006*, Červený Kláštor, Slovensko, 2006, pp. 125–130.
- [4] Fuxa, J., Fojtík, F., Kubala, R., Torque machine fit for high cycle fatigue of material testing, In: *Experimental Stress Analysis 2007*, Hotel Výhledy. 2007.
- [5] J. Fuxa, R. Kubala, F. Fojtík, Z. Poruba, Reconstruction of the Torque Test Machine and Its Utilization for Strength Criterion Searching In: *Experimental Stress Analysis 2005*, Skalský dvůr, 2005.