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Structural dynamic modification of vibrating systems

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Abstract

Vibration and acoustic requirements are becoming increasingly important in the design of mechanical structures. The need to vary the structural behaviour to solve noise and vibration problems occurs at design or prototype stage, giving rise to the so-called structural modification problem. Structural dynamic modification (SDM) as an application of modal analysis is a technique to study the effect of physical and geometrical parameter changes of a structural system on its dynamic properties which are mainly in the forms of natural frequencies and mode shapes. The fundamental approaches and formulations to SDM of vibrating systems are introduced. The changes of dynamical behaviour of a structure by modification of mass, damping and stiffness parameters of the structure are presented. The SDM of the real engineering structures is demonstrated. For these structures, it is more important to determine the structural modification in terms of physical and geometrical parameter changes related to mass, damping and stiffness parameters of vibrating structure. In this paper, the design and technological treatments are considered to achieve suitable vibration and acoustical properties of vibrating system. The modal properties of selected structures under physical parameters modification are studied.

Keywords: structural dynamic modification, modal analysis, damping, natural frequency, mode shape

1. Introduction

Structural modification within the frame of vibration analysis technology refers to a technique to modify physical properties of a structure in order to change or optimize its dynamic properties. Enhancement of the structural or acoustic response is one of the common goals of structural modification processes and can be related to any of the following elements: the source, the transmission path or the noise radiating component. This differs from structural modification for static analysis, where the changes are made to satisfy criteria of static design such as the reduction of stress concentration.

The dynamic characteristics of a structure usually referred to as its modal properties - natural frequencies and mode shapes, are determined by its mass, stiffness and damping distributions. The properties outlined by these distributions are called the spatial properties of the structure. The spatial properties are often quantified by a mathematical model of structure, such as a finite element (FE) model. This model translates the physical properties of the structure, such as its geometrical parameters and material properties, into distributed mass, stiffness and damping properties. For structural modification using an FE model, it is possible to determine the modification in terms of mass, stiffness and damping changes. However, for a real-life structure, it is more important to determine the structural modification in terms of geometrical parameter (such as thickness, length, diameter, etc.) changes or material property (such as damping coefficient, density, Young modulus, etc.) changes.

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There are two main reasons for structural modification. First, an existing structure may exhibit unsatisfactory dynamic characteristics. This is not unusual, since it is customary for design engineers to consider static loading and balance when designing a structure or a component, even if the structure may eventually work in an environment with dynamic loading. As we all are aware, a dynamic response can be many times greater than the static one, causing an excessive dynamic stress concentration that dwarfs any static stress concentration. Second, the design of a structure which is known to experience a dynamic working environment needs to satisfy some defined criteria such as averting vibration resonances. Generally, it can be said that the main objective of SDM techniques is to reduce vibration levels, shift resonances [8], improve dynamic stability, place optimally the modal points, perform modal synthesis, and optimize the weight and cost subject to dynamic constraints.

With exceptions, theoretically any physical or spatial change of a structure should result in modal property changes. This implies that for a proposed modal property change, almost any structural change of sufficient extent may realize it. Therefore, it seems to have numerous possible solutions of physical changes for a structural modification problem. The reality is different from this optimistic notion. Firstly, practical design and performance requirements often impose restrictions on structural modification. There are only limited modifications that are practical or feasible. Secondly, for a simple proposition of modal property change such as a natural frequency change to avert excitation frequency range, there are restrictions on changes brought unintentionally on other modal properties. Thirdly, other physical constraints, such as minimum total mass change to the structure, may be required. Many practical structural modification cases have very simple objectives. Most of the structural modification methods [1-3], [6], [7], [9] aim to change a structure's modal properties by modifying the existing physical or spatial properties. However, structural modification can also be carried out by adding a substructure to the original structure.

Two different problems are usually considered for structural modification - the *direct problem* and the *inverse problem*. The direct problem consists in determining the effect of already established modifications. This is a verification problem aimed at establishing the efficiency of performed changes on the dynamic behaviour of the considered system.

The inverse problem identifies, in the framework of a given set of possible modifications, the most appropriate changes required to obtain the desired dynamic behaviour. Therefore this is a typical design problem, consisting in specifying the required tools to obtain an improvement of the structural or acoustic response. The properties describing the original vibrating structure, whose dynamic behaviour is modified, are obtained either from an experimental database or from theoretical analysis using a finite element model of the system.

The problem of SDM techniques is very wide. The assumption of this article is to present a short summary of the structural dynamic modification techniques and the general mathematical theory of the modification process. The structural modifications of selected real-life structures are presented. The changes of modal properties of modified structures are studied in dependency on changes of spatial and physical properties.

2. Theoretical formulation of SDM

2.1. Vibrating systems with proportional damping

The dynamic behaviour of a proportionally damped structure which is assumed to be linear and discretized for n degrees of freedom can be described by the equations of motion

$$\mathbf{M}\ddot{\mathbf{x}} + \mathbf{B}\dot{\mathbf{x}} + \mathbf{K}\mathbf{x} = \mathbf{f}\ , \tag{1}$$

where **M**, $\mathbf{B} = \alpha\mathbf{M} + \beta\mathbf{K}$ and **K** are mass, damping and stiffness matrices, $\ddot{\mathbf{x}}, \dot{\mathbf{x}}$ and $\mathbf{x}$ are acceleration, velocity and displacement vectors of the structural points and $\mathbf{f}$ is force vector.

The undamped homogeneous equation

$$\mathbf{M}\ddot{\mathbf{x}} + \mathbf{K}\mathbf{x} = \mathbf{0}, \tag{2}$$

provides the eigenvalue problem

$$(\mathbf{K} - \lambda\mathbf{M})\boldsymbol{\phi} = \mathbf{0}. \tag{3}$$

The solution of (3) yields the matrices eigenvalues $\boldsymbol{\Lambda}$ and eigenvectors $\boldsymbol{\Phi}$

$$\boldsymbol{\Lambda} = \begin{bmatrix} \omega_1^2 & & \\ & \ddots & \\ & & \omega_n^2 \end{bmatrix}, \qquad \boldsymbol{\Phi} = [\boldsymbol{\phi}_1, \boldsymbol{\phi}_2, \dots, \boldsymbol{\phi}_n]. \tag{4}$$

The eigenvectors satisfy the orthonormal conditions

$$\boldsymbol{\Phi}^T\mathbf{M}\boldsymbol{\Phi} = \mathbf{I}, \quad \boldsymbol{\Phi}^T\mathbf{K}\boldsymbol{\Phi} = \boldsymbol{\Lambda}, \quad \boldsymbol{\Phi}^T\mathbf{B}\boldsymbol{\Phi} = \alpha\mathbf{I} + \beta\boldsymbol{\Lambda} = \boldsymbol{\Sigma}, \tag{5}$$

Using the transformation $\mathbf{x} = \boldsymbol{\Phi}\mathbf{q}$ in the equation of motion (1), and premultiplying by $\boldsymbol{\Phi}^T$ one obtains

$$\boldsymbol{\Phi}^T\mathbf{M}\boldsymbol{\Phi}\ddot{\mathbf{q}} + \boldsymbol{\Phi}^T\mathbf{B}\boldsymbol{\Phi}\dot{\mathbf{q}} + \boldsymbol{\Phi}^T\mathbf{K}\boldsymbol{\Phi}\mathbf{q} = \boldsymbol{\Phi}^T\mathbf{f} \tag{6}$$

and after arrangement

$$\ddot{\mathbf{q}} + \boldsymbol{\Sigma}\dot{\mathbf{q}} + \boldsymbol{\Lambda}\mathbf{q} = \boldsymbol{\Phi}^T\mathbf{f}. \tag{7}$$

The structural modifications cause changes in the parameter matrices of the spatial model of the structure and the above equations get modified as

$$(\mathbf{M} + \Delta\mathbf{M})\ddot{\mathbf{x}} + (\mathbf{B} + \Delta\mathbf{B})\dot{\mathbf{x}} + (\mathbf{K} + \Delta\mathbf{K})\mathbf{x} = \mathbf{f}, \tag{8}$$

where $\Delta\mathbf{M}$, $\Delta\mathbf{B}$ and $\Delta\mathbf{K}$ are changes in the parametric matrices due to modification.

Using again the coordinate transformation $\mathbf{x} = \boldsymbol{\Phi}\mathbf{q}$, premultiplying equation (8) by $\boldsymbol{\Phi}^T$ and after arrangement, we obtain

$$(\mathbf{I} + \boldsymbol{\Phi}^T\Delta\mathbf{M}\boldsymbol{\Phi})\ddot{\mathbf{q}} + (\boldsymbol{\Sigma} + \boldsymbol{\Phi}^T\Delta\mathbf{B}\boldsymbol{\Phi})\dot{\mathbf{q}} + (\boldsymbol{\Lambda} + \boldsymbol{\Phi}^T\Delta\mathbf{K}\boldsymbol{\Phi})\mathbf{q} = \boldsymbol{\Phi}^T\mathbf{f}. \tag{9}$$

It is important to note, that the matrices

$$\Delta\hat{\mathbf{M}} = \boldsymbol{\Phi}^T\Delta\mathbf{M}\boldsymbol{\Phi}, \qquad \Delta\hat{\mathbf{B}} = \boldsymbol{\Phi}^T\Delta\mathbf{B}\boldsymbol{\Phi}, \qquad \Delta\hat{\mathbf{K}} = \boldsymbol{\Phi}^T\Delta\mathbf{K}\boldsymbol{\Phi}, \tag{10}$$

are not usually diagonalised by the eigenvectors of the original structure. Then, the equations of motion expressed in the modal coordinates of the original structure are coupled

$$(\mathbf{I} + \Delta\hat{\mathbf{M}})\ddot{\mathbf{q}} + (\boldsymbol{\Sigma} + \Delta\hat{\mathbf{B}})\dot{\mathbf{q}} + (\boldsymbol{\Lambda} + \Delta\hat{\mathbf{K}})\mathbf{q} = \boldsymbol{\Phi}^T\mathbf{f}. \tag{11}$$

From this reason, it is required to study undamped homogenous problem

$$(\mathbf{I} + \Delta\hat{\mathbf{M}})\ddot{\mathbf{q}} + (\boldsymbol{\Lambda} + \Delta\hat{\mathbf{K}})\mathbf{q} = \mathbf{0} \tag{12}$$

and to solve the eigenvalue problem of undamped modified structure in the form

$$[(\boldsymbol{\Lambda} + \Delta\hat{\mathbf{K}}) - \lambda_m(\mathbf{I} + \Delta\hat{\mathbf{M}})]\boldsymbol{\phi}_m = \mathbf{0}. \tag{13}$$

Using the new transformations $\mathbf{q} = \mathbf{\Phi}_m \mathbf{q}_m$ and premultiplying by $\mathbf{\Phi}_m^T$, the equations of motion are uncoupled, the new eigenvalues $\mathbf{\Lambda}_m$ and the new eigenvectors $\mathbf{\Phi}_m$ are determined and following ortonormal conditions are held

$$\mathbf{\Phi}_m^T(\mathbf{I} + \Delta\hat{\mathbf{M}})\mathbf{\Phi}_m = \mathbf{I}, \quad \mathbf{\Phi}_m^T(\mathbf{\Sigma} + \Delta\hat{\mathbf{B}})\mathbf{\Phi}_m = \mathbf{\Sigma}_m, \quad \mathbf{\Phi}_m^T(\mathbf{\Lambda} + \Delta\hat{\mathbf{K}})\mathbf{\Phi}_m = \mathbf{\Lambda}_m. \tag{14}$$

Then, the uncoupled form of equations motion (11) has the form

$$\ddot{\mathbf{q}}_m + \mathbf{\Sigma}_m \dot{\mathbf{q}}_m + \mathbf{\Lambda}_m \mathbf{q}_m = \mathbf{\Phi}_m^T \mathbf{\Phi}^T \mathbf{f}. \tag{15}$$

The natural frequencies of the modified structure can be obtained from the eigenvalue matrix $\mathbf{\Lambda}_m$. The new mode shapes of the modified structure are a linear combination of the original structure and they are generally expressed by $\mathbf{\Psi} = \mathbf{\Phi}\mathbf{\Phi}_m$. Then the physical coordinates can be written as $\mathbf{x} = \mathbf{\Phi}\mathbf{q} = \mathbf{\Psi}\mathbf{q}_m = \mathbf{\Phi}\mathbf{\Phi}_m \mathbf{q}_m$.

2.2. Vibrating systems with hysteretical damping

The dynamic behaviour of linear multi-degrees of freedom structure with structural (hysteretic) damping can be described by the equation of motion

$$\mathbf{M}\ddot{\mathbf{x}} + (\mathbf{K} + i\mathbf{H})\mathbf{x} = \mathbf{f} \tag{16}$$

where **M** is the mass matrix, **H** is the hysteretic damping matrix, **K** is the stiffness matrix of the system. The vectors $\mathbf{x}$, $\ddot{\mathbf{x}}$ and **f** are the displacements, accelerations and forces respectively and $i = \sqrt{-1}$ is imaginary unit.

Phenomenologically, the hysteretic damping is defined on the base of harmonic motion of the structure. Then, for harmonic excitation, it is necessary to consider that the force vector **f** and the displacement vector **x** are as follows

$$\mathbf{f} = \mathbf{F}e^{i\omega t}, \quad \mathbf{x} = \mathbf{X}e^{i\omega t}. \tag{17}$$

After substituting (17) into (17), the equation of motion has the form

$$[(\mathbf{K} - \omega^2\mathbf{M}) + i\mathbf{H}]\mathbf{X} = \mathbf{F}, \tag{18}$$

or

$$\mathbf{D}(\omega)\mathbf{X} = \mathbf{F}, \tag{19}$$

where $\mathbf{D}(\omega) = \mathbf{K} - \omega^2\mathbf{M} + i\mathbf{H}$ can be defined as a dynamic stiffness matrix (DSM).

If a modification in the hysteretically damped structure has to be incorporated through changes in its parameters, the equation (18) can be rewritten as

$$[(\mathbf{K} + \Delta\mathbf{K}) - \omega^2(\mathbf{M} + \Delta\mathbf{M}) + i(\mathbf{H} + \Delta\mathbf{H})]\mathbf{X}_m = \mathbf{F}, \tag{20}$$

where $\Delta\mathbf{M}$, $\Delta\mathbf{H}$ and $\Delta\mathbf{K}$ are changes in the parametric matrices, $\mathbf{X}_m$ is new vector of displacement of the modified structure.

The equation (20) can be arranged as follows

$$[(\mathbf{K} - \omega^2\mathbf{M} + i\mathbf{H}) + (\Delta\mathbf{K} - \omega^2\Delta\mathbf{M} + \Delta\mathbf{H})]\mathbf{X}_m = \mathbf{F}, \tag{21}$$

or

$$[\mathbf{D}(\omega) + \Delta\mathbf{D}(\omega)]\mathbf{X}_m = \mathbf{D}_m(\omega)\mathbf{X}_m = \mathbf{F}, \tag{22}$$

where

$$\Delta\mathbf{D}(\omega) = \Delta\mathbf{K} - \omega^2\Delta\mathbf{M} + \Delta\mathbf{H}\,, \tag{23}$$

is the change in DSM due to the changes in geometrical parameters and material properties and $\mathbf{D}(\omega)$ is DSM of the modified structure.

From equation (23) it is clear that the construction of modifying DSM ($\Delta\mathbf{D}(\omega)$), for the modifying element, is required. It is also obvious that the modifying DSM should be such that it can be directly added to the unmodified DSM ($\mathbf{D}(\omega)$) to get the modified DSM ($\mathbf{D}_m(\omega)$). The construction of the modifying DSM depends on specific possibilities different modifying elements – pure mass modification, pure stiffness modification, pure damping modification or mutual combination of these modifications.

Generally, the extraction of the natural frequencies and related mode shapes can be performed using real part of the complex DSM (18), i.e.

$$\mathbf{R}(\omega) = \mathbf{K} - \omega^2\mathbf{M}\,. \tag{24}$$

The matrix $\mathbf{R}(\omega)$ is a function of the forcing frequency ω. The determination of natural frequencies and mode shapes corresponds to the solution of eigenvalue problem (3). Using this procedure required set of natural frequencies and related mode shapes are determined.

3. Numerical examples

3.1 Structural modification of cantilever beam by the constraining viscoelastic layers

The dynamical properties of cantilever beam embedded into constraining layers (Fig. 1) are studied. For this structure the elastic properties of beam element are considered and for isotropic linear viscoelastic material of constraining layers, energy dissipation under harmonic vibration is taken into account with complex Young modulus of elasticity.

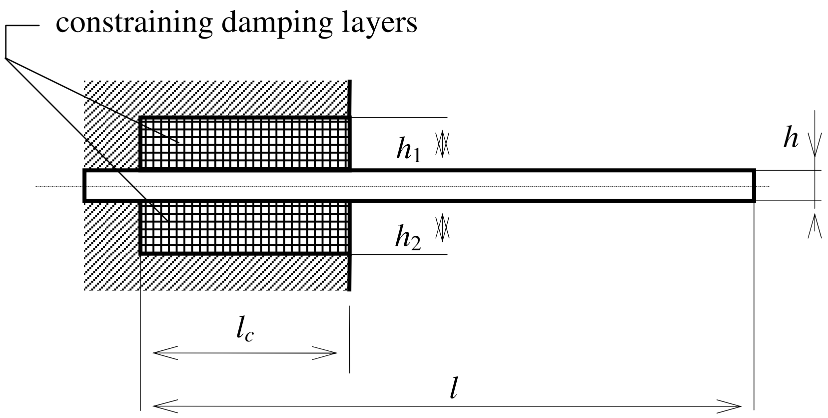

Fig. 1. Cantilever beam partially embedded into constraining layers.

Finite element formulation for beam element embedded into constraint damping layers was carried out following the standard procedure of finite element method. The element matrices for this type of structure were derived in detail in [5].

Let us consider an clamped beam uniform thickness h, width b and length L, partially embedded into constraint layers. The beam is bilaterally covered by constraining layers uniform thickness h_1, resp. h_2 which are embedded into foundation.

The main assumptions we make are concerned with the cross sectional behaviour of the beam, resp. cross sectional behaviour of constraint layers during deformation. The behaviour of cross section of beam is assumed in framework of classical beam theory. The point displacements of constraining layers are assumed as linear functions of point position of constraining layer to the reference plane [5]. The contact between structural element and constraint layers is assumed without friction. The assumption of perfect adhesion of layers and structural element is considered, i.e. no relative motion of interface points are occurred during deformation of structure. The normal to the neutral axis undeformed structure represented by a straight line could be for deformed structure represented by a set of three straight lines

The total strain energy of structure is supposed in the form

$$U = U_b + \sum_{j=1}^{2} U_{bj} = \frac{1}{2}\left\{ \int_{\Omega_b} \boldsymbol{\sigma}_b^T \boldsymbol{\varepsilon}_b \mathrm{d}\Omega_b + \sum_{j=1}^{2} \int_{\Omega_{bj}} \boldsymbol{\sigma}_{bj}^T \boldsymbol{\varepsilon}_{bj} \mathrm{d}\Omega_{bj} \right\}, \tag{25}$$

where Ω_b is domain occupied by beam, Ω_{bj} is domain occupied by jth constraining layer.

The total kinetic energy is expressed as

$$T = T_b + \sum_{j=1}^{2} T_{bj} = \frac{1}{2}\left\{ \int_{\Omega_b} \rho_b \dot{\mathbf{u}}_b^T \dot{\mathbf{u}}_b \mathrm{d}\Omega_b + \sum_{j=1}^{2} \int_{\Omega_{bj}} \rho_j \dot{\mathbf{u}}_{bj}^T \dot{\mathbf{u}}_{bj} \mathrm{d}\Omega_{bj} \right\}, \tag{26}$$

where ρ_b, resp. ρ_{bj} are density of the beam, resp. density of the constraining layers and $\dot{\mathbf{u}}_b$, resp. $\dot{\mathbf{u}}_{bj}$ are velocities of the beam points, resp. velocities of the constraining layer points.

The work of external forces is defined by

$$W = \int_{\Gamma_{tp}} \mathbf{p}^T \mathbf{q}_b \mathrm{d}\Gamma_{tp}, \tag{27}$$

where Γ_{tp} is the portion of the boundary where the tractions are described and $\mathbf{p}^T$ is a vector of external surface distributed load and $\mathbf{q}_b$ is displacement vector of points on the neutral axis.

To develop the equations of motion of a beam element embedded into constraint layers, Hamilton's variational principle is used. According to Hamilton's variational principle, the first variation must be fulfilled

$$\int_{t_1}^{t_2} (\delta T - \delta U + \delta W)\mathrm{d}t = 0. \tag{28}$$

The hysteretic damping in material of the constraining layers is considered. The damping properties of the constraining layers are taken into account in the form of complex modulus

$$E^c = E^r + iE^i = E(1 + \mathrm{i}\beta), \tag{29}$$

where $E^R = E$ is real part of complex modulus - Young modulus, E^I is imaginary part of complex modulus (loss modulus) and $\beta = E^I / E^R$ is material loss factor.

Using the hysteretic damping model defined above in material of constraining layers, we obtain the finite element equations of motion in the matrix form

$$(\mathbf{M}_b + \Delta\mathbf{M}_l)\ddot{\mathbf{q}}^c + [(\mathbf{K}_b + \Delta\mathbf{K}_l) + i\Delta\mathbf{H}_l]\mathbf{q}^c = \mathbf{f}\,, \tag{30}$$

where $\mathbf{M}_b$ and $\mathbf{K}_b$ are the mass and stiffness matrix of the beam without constraining layers, $\Delta\mathbf{M}_b$ and $\Delta\mathbf{K}_b$ are the changes of the mass and stiffness structural matrix caused by constraining layers and $\Delta\mathbf{H}_l$ is the hysteretic matrix considering the hysteretic damping of constraining layers. The vector of nodal displacement $\mathbf{q}^c$ is also complex.

The equation (30) is used to finite element formulation of the eigenvalues problem. The damping model of constraining layers is defined only for structures under harmonic vibrations (17). The vector of nodal displacements and force vector is supposed in the form

$$\mathbf{q}^c = \mathbf{X}^c e^{i\omega t}\,, \qquad \mathbf{f} = \mathbf{F}e^{i\omega t}\,, \tag{31}$$

where $\mathbf{X}^c = \mathbf{X}^r + i\mathbf{X}^i$ is complex eigenvector, while $\mathbf{X}^r$ and $\mathbf{X}^i$ are real part and imaginary parts of complex eigenvectors, respectively.

Then, eigenvalue problem of the equation of motion (30) can be written in the following matrix form

$$[(\mathbf{K}_b + \Delta\mathbf{K}_l) - \lambda_i^c(\mathbf{M}_b + \Delta\mathbf{M}_l) + i\Delta\mathbf{H}_l]\mathbf{X}_i^c = \mathbf{0}\,, \tag{32}$$

where λ_i^c is i-th complex eigenvalues of the structure and $\mathbf{X}_i^c$ is related complex eigenvector.

Equation (32) gives the i-th natural frequency ω_i and i-th modal loss factor η_i for mode as follows

$$\omega_i = \sqrt{\mathrm{Re}[\lambda_i^c]}\,, \qquad \eta_i = \frac{\mathrm{Im}[\lambda_i^c]}{\mathrm{Re}[\lambda_i^c]} = \frac{\lambda_i^i}{\lambda_i^r}\,, \tag{33}$$

where $\lambda_i^c = \lambda_i^r + i\lambda_i^i = \omega_i^2(1 + \mathrm{i}\eta_i)$ is complex eigenvalue, $\lambda_i^r = \mathrm{Re}[\lambda_i^c]$ and $\lambda_i^i = \mathrm{Im}[\lambda_i^c]$ are real and imaginary parts of i-th complex eigenvalue, respectively.

After introducing of the following non-dimensional parameters:

- non-dimensional density of j-th constraining layer $\bar{\rho}_j = \dfrac{\rho_j}{\rho}$,
- non-dimensional Young modulus of j-th constraining layer $\bar{E}_j = \dfrac{E_j}{E}$,
- non-dimensional thickness of j-th constraining layer $\bar{h}_j = \dfrac{h_j}{h}$,
- non-dimensional length of layers $\bar{l} = \dfrac{l_c}{l_0}$,

the natural frequencies and loss factors of the cantilever beam (Fig. 1) which is embedded into constraining damping layers are determined.

The objective of this section is to present some interesting results obtained by numerical analysis of beam embedded into viscoelastic environment. The natural frequency and modal loss factor of this structure are in the spotlight of presented numerical analysis. In order to provide integrate summary about properties of beam structure created using this manner, the dependencies of natural frequencies and modal loss factors *vs.* geometrical parameters and material properties are presented. The following parametric studies show the influence of non-dimensional parameters on the resonance frequency and modal loss factor of beam structure. The considered material properties and geometry parameters of the beam structure (Fig. 1.) are as follows:

- beam structural element
  E = 70 GPa; ρ = 2700 kgm^{-3}; h = 0,01 m; l = 0,3 m; b = 0,05 m;
- constraining layers (the same material properties and geometry for both layers)
  $E_1 = E_2$ = 0,07 GPa; $\rho_1 = \rho_2$ = 1000 kgm^{-3}; $h_1 = h_2$ = 0,01÷ 0,1 m; l = 0 ÷ 0,3 m; b = 0,05 m;
  $\beta_1 = \beta_2 = 0{,}2$

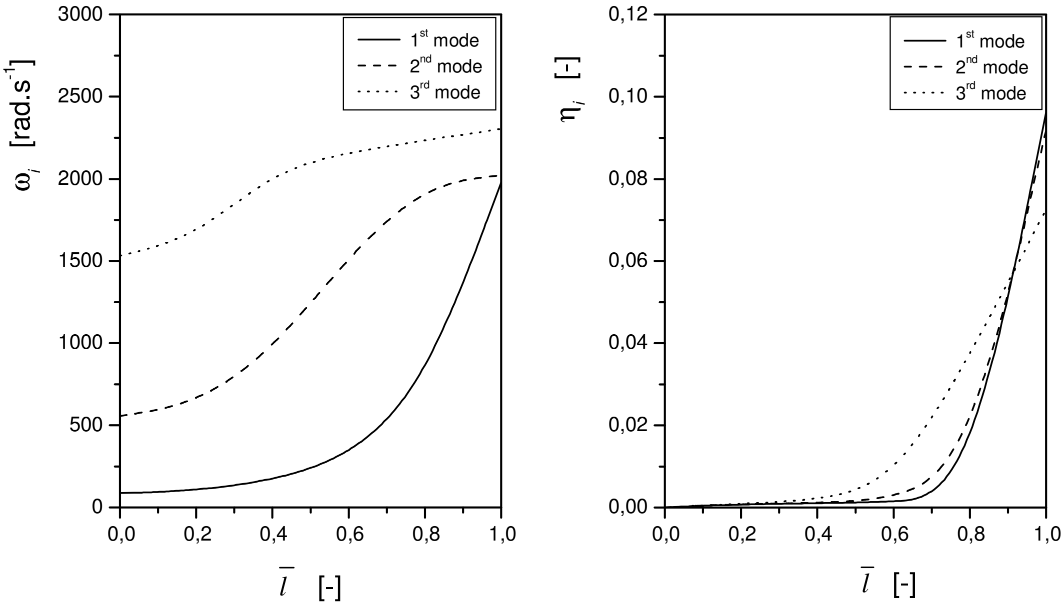

Fig. 2. Dependencies of the natural frequency ω_i and modal loss factor η_i of the first three modes on non-dimensional length of layers $\bar{l}$ (for $\bar{h} = 2{,}5$).

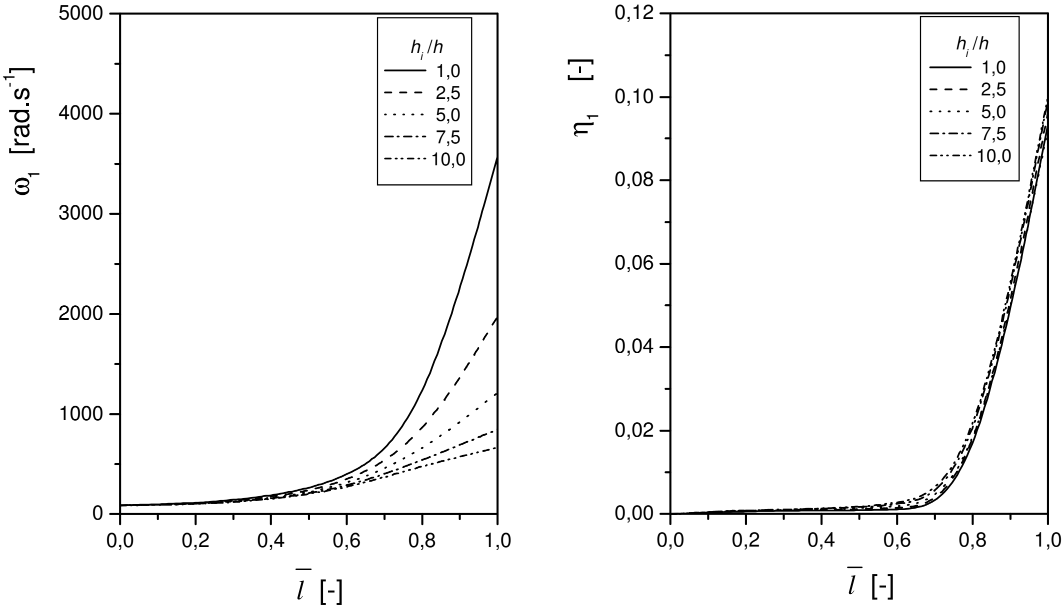

Fig. 3. Dependencies of the first natural frequency ω_1 and the first modal loss factor η_1 on non-dimensional length of layers $\bar{l}$ and for different $\bar{h}$.

The effects of the geometrical parameters and material properties of the structure on the natural frequencies and modal loss factors of beam structure are studied. The results show that the geometrical parameters and material properties of the layers have very important effect on dynamical characteristics. It can be said that the structural mass and stiffness properties depending on parameters of layers ($\bar{l}$ and $\bar{h}$) have the influence on the resonance frequency and modal loss factor of structure.

This example presents a suitable manner to change of the modal properties by introduced structural modification of beam structure. These formulations and results can be used to vibrational tuning of hysteretically damped mechanical systems as well as a starting point for determination of required quantity of modal loss factor. This approach to modelling of mechanical structures provides the possibility to create of effectively damped mechanical structures.

3.2 Structural modification of circular disc by in-plane residual stresses

The circular discs are structural elements widely used in the structural and processing applications. One of the most used geometric shapes for processing and cutting operation material is thin circular disk - circular saw blade. Circular saw blades are widely used for cutting and forming metal and non-metal materials. Dynamical properties of circular saw blade are investigated in this part.

One of the techniques of the structural modifications of disc to achieve the required dynamic properties is to initiate pre-stress in disc plane. This is concerned primarily with "tuning" of dynamical properties of vibrating circular discs by technological treatments inducing the residual in-plane stresses. There are more possibilities to obtain the disc in-plane stresses. One of them is rolling of disc surface. In the roll-tensioning process, the disc is compressed within a certain annular contact zone between two opposing rollers. Plastic deformations in the contact zone of circular disc result residual in-plane stresses in whole disc.

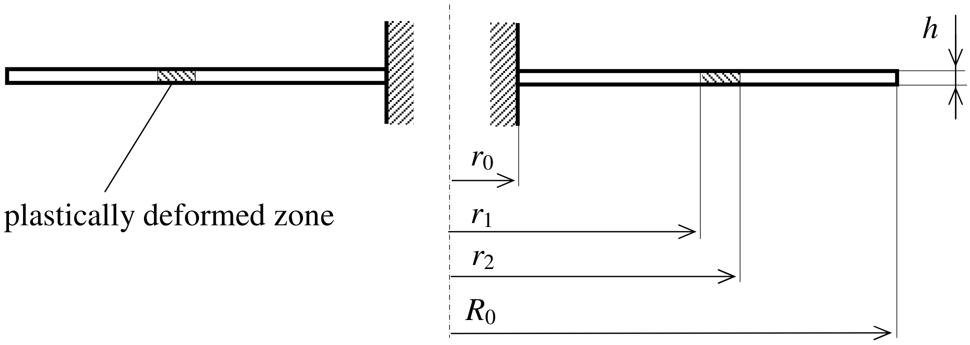

Fig. 4. *The circular disc with plastically deformed zone.*

The effects of residual stresses induced by roll-tensioning on dynamical properties are analysed. The natural frequency characteristics for various rolling position and various rolling depth of the annulus are obtained by modal analysis using finite element method (FEM). The role of residual stresses obtained by rolling is assessed from the changes in natural frequencies and modal shapes.

Generally, the stress-strain relations with initial stress and initial strain are given by

$$\boldsymbol{\sigma} = \mathbf{D}(\boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}_0) + \boldsymbol{\sigma}_0 , \qquad (34)$$

where $\boldsymbol{\sigma}$ and $\boldsymbol{\varepsilon}$ are stress and strain vector; $\boldsymbol{\sigma}_0$ and $\boldsymbol{\varepsilon}_0$ are initial stress and initial strain vector; $\mathbf{D}$ is elasticity matrix.

Using the finite element formulation, the equation of motion for a free vibration of disc without of in-plane stressed disc is described by expression

$$\mathbf{M}\ddot{\mathbf{x}} + \mathbf{K}\mathbf{x} = \mathbf{0}, \tag{35}$$

and after structural modification (with in-plane residual stress) made by rolling

$$\mathbf{M}\ddot{\mathbf{x}} + (\mathbf{K} + \Delta\mathbf{K}_{\sigma})\mathbf{x} = \mathbf{0}, \tag{36}$$

where $\mathbf{M}$ is mass matrix; $\mathbf{K}$ is stiffness matrix, $\Delta\mathbf{K}_{\sigma}$ is the change of stiffness matrix following from stress distribution induced by rolling; $\ddot{\mathbf{x}}$ and $\mathbf{x}$ are nodal accelerations and nodal displacements.

From equation (35) follows that after rolling the mass distribution is not changed, but stiffness characteristics vary greatly. The natural frequencies and modal shapes of a circular disc with rolling induced residual stress distribution can be obtain from the following eigenvalue problem

$$(\mathbf{K} + \Delta\mathbf{K}_{\sigma} - \omega^2\mathbf{M})\mathbf{q} = \mathbf{0}, \tag{36}$$

where ω is eigenvalue frequency.

In order to calculate the variation of disc stiffness $\Delta\mathbf{K}_{\sigma}$ after rolling, we must know the residual stress distribution in a disc plane. To determine the residual stress distribution, the method of thermal stress loading is used [4]. The thermal expansion induces a stress distribution, which is analogous to the stress distribution initiated by rolling. The dependence between temperature and depth of roll-tensioning is approximately described by equation

$$\Delta T \approx \frac{\mu}{h\alpha}\Delta z, \tag{37}$$

where μ is Poisson number, α is temperature expansion coefficient, h is disc thickness and Δz is depth of roll-tensioning.

The natural angular frequencies and modal shapes of circular disc with residual stress distribution for different position, depth and width of roll-tensioning can be determined from equation (36). To obtain appropriate dynamical properties of circular disc, it is necessary to determine the natural frequency curves for the various position of mean radius of the roll-tensioning annulus $r_c = (r_2 + r_1)/2$.

We consider a circular disc (Fig. 4.) of the outer radius R_0 = 120 mm, inner radius (flange radius) r_0 = 25 mm, thickness h = 1,8 mm. The width of plastically deformed annulus is assumed $r_2 - r_1$ = 10 mm. This width was selected arbitrarily and it was considered as a representative value for planned experimental verification of investigated phenomenon. The disc is assumed to be perfectly fixed in region $r \le r_0$. The outer edge of circular disc is free.

The influence of mean rolling radius r_c and depth of rolling Δz on the natural frequencies for mode shapes 0/1, 0/0, 0/2, 0/3 (number of nodal circles/number of nodal lines) is presented on Fig. 5. The trend of frequency curves for modal shapes 0/1 and 0/0 differs from the trend of frequency curves for modal shapes 0/2 and 0/3. The natural frequencies of the modal shapes 0/2 and 0/3 increase with r_c until the maximum values near $r_c \approx 60$ mm. For radius $r_c > 60$ mm, they decrease. Contrary to this, the natural frequencies of the modal shapes 0/1 and 0/0 are decreasing with r_c and for $r_c \approx 50$ mm reach the minimum; then they increase. The influence of mean rolling radius r_c and depth of rolling Δz of plastically deformed annulus on the natural frequencies for individual modal shapes is shown on Fig. 6.

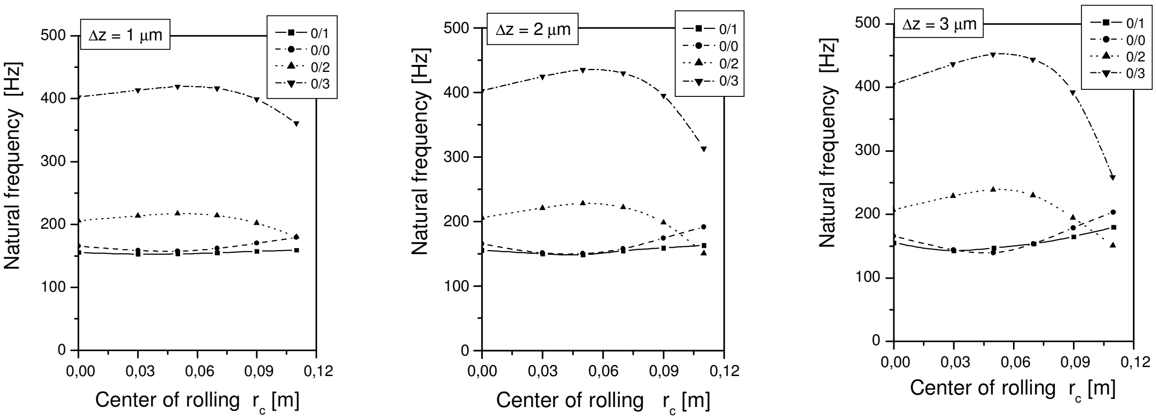

Fig. 5. Effect of rolling depth Δz and mean rolling radius r_c on the first four natural frequencies.

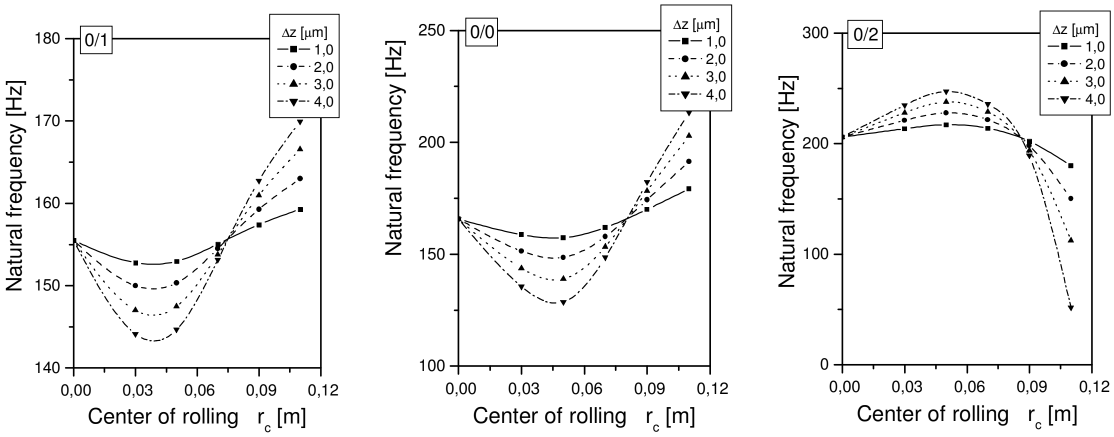

Fig. 6. Effect of rolling radius r_c and rolling depth Δz on the natural frequencies (mode shapes 0/1, 0/0, 0/2).

This manner of structural modification of circular disc provides very effective tool to structural dynamic modification of such structures for which the modification of geometrical parameters, connection of mass or stiffness elements is not possible.

4. Conclusion

Structural modification as a technique to change the dynamic characteristics of a structure has been used by engineers for decades. The main objective of SDM techniques is to reduce vibration levels, shift resonances, improve dynamic stability, place optimally the modal points, perform modal synthesis, and optimize the weight and cost subject to dynamic constraints. Generally, the modification may be expressed in terms of the incremental mass, damping, and stiffness matrices.

In this paper, a short summary about theoretical principles of the structural dynamic modification of vibrating systems and the general mathematical theory of the modification process are presented. The solution procedures for structural modification problems of the vibrating systems with proportional and hysteretic damping are introduced.

Subsequently, the structural modifications of real-life structures are presented. The changes of modal properties of selected modified structures are studied in dependency on changes of spatial and physical properties. Two examples of modification of vibrating structures are selected – cantilever beam embedded into viscoelastic constraining layers a circular disc with in-plane residual stresses. The obtained results confirm that structural dynamic modification is very effective tool to change of dynamical properties vibrating systems.

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